A Multi-Objective Optimization Model to Plan City-Scale Water Systems with Economic and Environmental Objectives: A Case Study in Santiago, Chile

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Abstract

Climate Change and its effects in water scarcity has become an important challenge for cities with water management problems. These problems require an integral planning of the city, which can be supported by optimization. The main goal of the research is to provide a regional optimization model for water networks, including new treatment options. The model is formulated as a multi-objective mixed-integer programming problem, focused on environmental and economic impact of the network, minimizing water extracted from natural sources and total cost. The formulation is developed with the goal-programming methodology. The model covers a complete existing city-scale water network, including 4 different options of water reuse within the city: drinking water, fresh water, irrigation, and discharge in natural courses. The case study is Santiago, capital of Chile, which is the political, economic, and institutional center of Chile. If both objective functions have equal importance to configure the solution, the following ideas characterize the optimal water network: (i) it is more environmentally and economically convenient to reuse water within the network rather than recycling water to the natural source; (ii) the reuse of water is preferred in the form of irrigation and drinking qualities rather than industrial qualities to reduce transport costs, and (iii) the modification of the current treatment plants is preferred, because of the high cost of installation of new plants. An environmental and cost-effective solution for Santiago, Chile, can reduce the source water extraction in 35.7%. The model can be implemented in other contexts, providing orientations to decision-makers to plan city-scale water networks with simultaneous environmental and economic considerations.

Keywords: Water Networks ,Regional Integration ,Multi-Objective Optimization

1. Introduction

Climate change has become an important issue in several regions worldwide since human behavior is conditioned by the effects of Global Warming on Earth [1]. A direct effect is water scarcity [2], considered a global risk by the World Economic Forum [3]. To address this issue, long term planning of city-scale water systems has become a key matter. In this context, optimization techniques can be a valuable tool for water resource management in order to redesign regional water systems. This topic constitutes the focus of this project.

Some efforts have previously been made to model and optimize both Industrial Water Networks (IWN) and water networks within Eco-Industrial Parks (EIP), having environmental and economic benefits.

Campos de Faria et al. (2009) looked for the minimum operational cost and freshwater consumption of an IWN [4]. The results showed that it is useful to identify reuse opportunities. Boix et al. (2012) developed a multi-objective optimization model of an EIP [5]. Liu et al (2011) developed an optimization model for water resources management for insular areas in Greece [6]. Rojas-Torres et al. (2015) proposed a multi-objective optimization model to design a water system in a city scale, using water reuse for agricultural purpose [7]. Pérez et al. (2017) also proposed a model to design a water system in a city in Mexico but using rainwater reservoirs [8].

The objective when redesigning water systems is to sustainably improve the use of the resource. With this purpose, incorporating non-conventional water sources and the recycling wastewater have an important potential. Many efforts have been made to reuse and recycle water.

Lovelady and El-Halwagi (2009) developed a model to plan water management among multiple processes in a EIP facility [9]. Campos de Faria et al. (2009) proposed alternatives to optimize IWN using different regeneration units [4]. Sadegh et al. (2011) presented a model to minimize the energy of an inter-plant water network in an EIP [10].

Some studies refer to the incorporation of new sources in a city-scale optimization model. Liu et al (2011) presented an optimization approach for water management of a city including desalinated seawater and reclaimed water as water sources [6]. Rojas-Torres et al. (2015) incorporated rainwater harvesting and reclaimed water [7].

Other option is to retrofit existing water networks in order to redesign them. Campos de Faria et al. (2009) presented a methodology for retrofitting an IWN [4]. Sotelo-Pichardo et al. (2011) proposed a mathematical programming model for the optimal retrofitting of an IWN [11]. Rubio-Castro et al. (2012) developed a model to design an EIP by retrofitting existing water networks [12].

As cited above, research with plant modification is mainly applied in IWN. The present paper includes the possibility of installing new plants or retrofitting existing ones on a city scale.

Water system modification has different impacts. These impacts have been studied considering mainly economic or environmental dimensions.

Some authors focused on reducing the associated cost. Bagajewicz et al. (2000), focused on minimizing the operational and investment cost of a water network problem by using a tree search algorithm [13]; Liu et al. (2011) proposed a MILP problem by minimizing capital and operating costs applied to a city [6]. Finally, Rubio-Castro et al. (2012), proposed a

MINLP problem minimizing plant capital and piping operation costs, applied to an eco-industrial park[12].

On the other hand, other authors focus on the importance of the environmental impact generated by the water network. Boix et al. (2011) presents a MINLP problem applied to a IWN, where it seeks to minimize the flow of water extraction from natural sources [14]. Mughees et al. (2013) seeks to increase the water efficiency of a petrochemical plant by minimizing its water consumption through a MINLP problem formulation [15]. Finally, Hansen et al. (2018) also seeks to minimize the water consumption of a IWN by formulating an NLP problem [16].

Simultaneous minimization of environmental and economic impacts is less common. Kantor et al. 2015 developed a model to reduce network life cycle emissions while seeking to reduce their cost in an EIP network [17]. Rojas-Torres et al. (2015) proposed a model to solve planning and scheduling water for a city, maximizing overall profit and minimizing fresh water consumption and land use [7]. Pérez et al. (2019) proposed to design an optimal water distribution network maximizing revenues and minimizing both groundwater usage and investment cost [18].

The present research seeks to reduce the costs of installing treatment plants and operating the network, and to reduce the flowrate of extraction from natural sources, incorporating the economic and environmental impact.

None of the previously mentioned investigations present a multi-objective optimization problem on a city scale, considering the reuse of 4 different water qualities to supply different consumers. This way of approaching the problem, not considered before, is presented in this study. A novel superstructure is created for modeling a city-scale water system in order to plan new treatment plants and connections among stakeholders, taking into account simultaneous economic and environmental objectives.

The problem is formulated as a Multi-Objective Mixed-Integer Programming to decide the optimal configuration of a regional water system including environmental and economic considerations. The model is formulated to decide (i) the installation of new treatment plants, (ii) the actualization of the existing ones, and (iii) the connections within the new integration network. These changes on the water network allow to recycle and reuse water. The objective functions to minimize are the water usage from the source and the total cost of the water system. The problem is solved using the goal programming technique. The main novelties of this work are the large scale orientation of the formulation and the integration of economic and environmental objectives in the planning of a city-scale water system.

2. Problem Structure

The methodology used for this research is shown in Figure 1. Every step was followed by an analysis of the information or results.

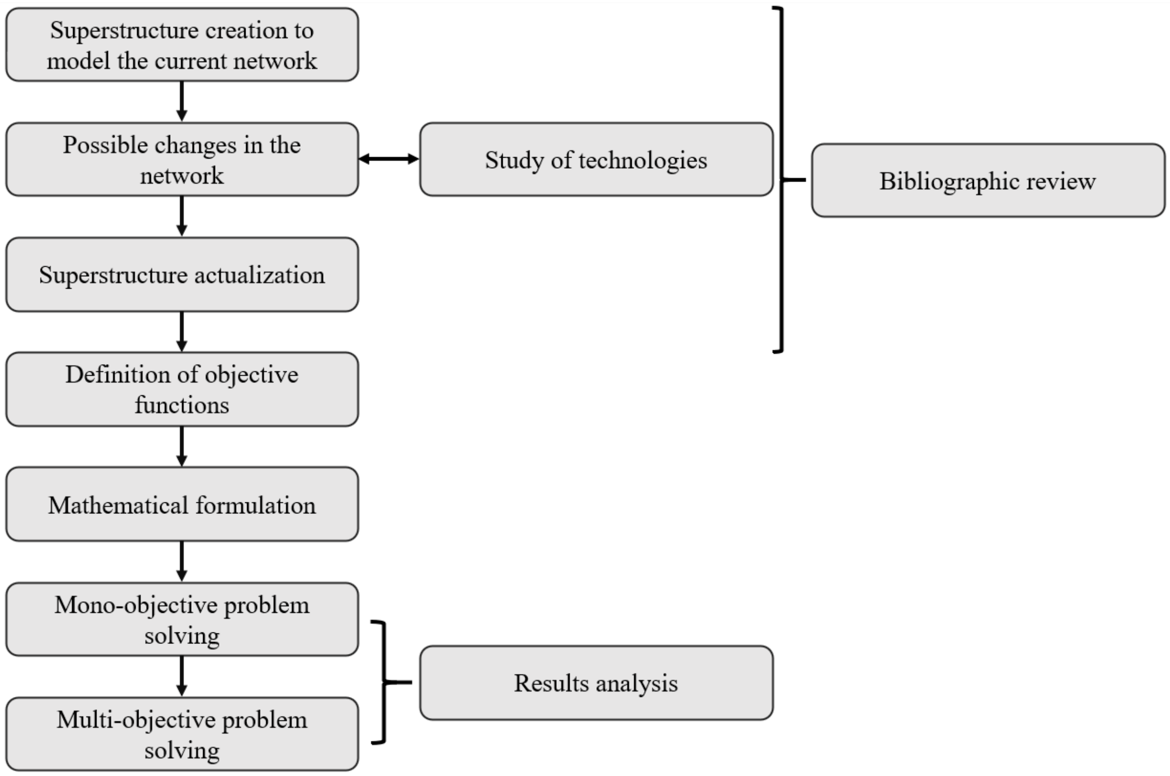


Figure 1: Diagram representing the methodology followed in the research.

In this problem, a city water system is modelled. This section describes the superstructure with all possible connections. The whole surface of the city is divided into sub-regions, based on population distribution. Participants are classified into (i) consumers; (ii) sources; (iii) distribution and collection nodes; (iv) treatment plants; and (v) final disposal sinks. Figure 2 shows the simplified graph.

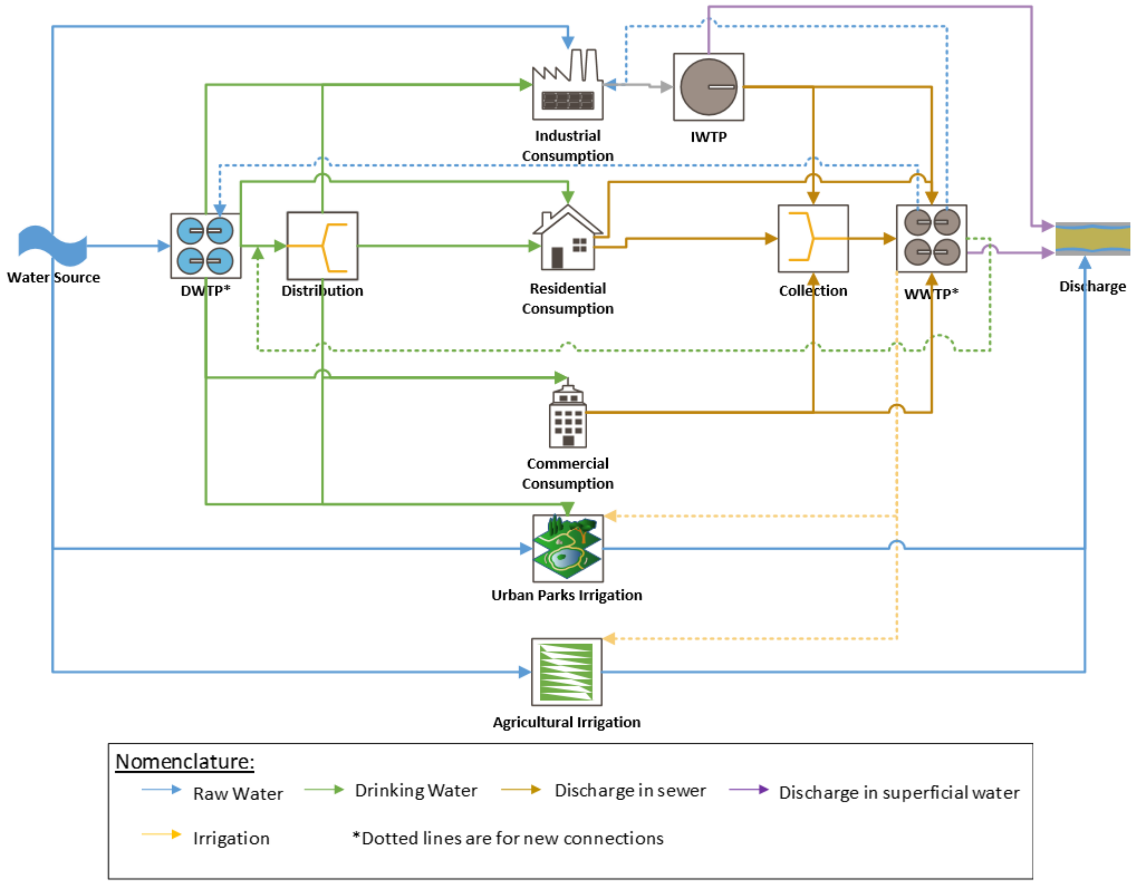


Figure 2: Graph of the problem. Current and new plants are included as treatment nodes, and the respective consumption within a city.

There are two subsets of water sources and disposal sinks: surface and underground water. Consumption nodes represent a variety of uses: (i) domestic, (ii) commercial, (iii) industrial, (iv) agricultural, and (v) irrigation of urban areas. Each consumption node has its own demand. Some consumers require less restrictive qualities than drinking water.

Industrial consumption is subdivided in two subsets, depending on consumption magnitude. The first subset is large industrial consumers, which are particular big industries in the city. The second subset group, other companies represented as a cluster within each sub-region.

Concerning treatment nodes, different sets are created for existing and new treatment plants. Existing plants are in their original location. Also, all existing wastewater treatment plants have discharge quality water. Therefore, there is no recycling or reuse of water in the present system.

Four new sets of new wastewater treatment plants with different technological configurations were created. These can achieve four water qualities: (i) freshwater; (ii) drinking water; (iii) irrigation water; and (iv) discharge water. In addition, three new sets of modified treatment plants are created. Modified plants are existing plants modified to achieve a different output quality. These plants can achieve irrigation, water source, and drinking water quality.

DWTP and WWTP are subdivided into two subsets depending on their treatment capacity: large and small plants. Distribution and collection nodes aim at representing water distribution and collection networks of the current system. There is also a sink node to collect lost water from the nodes within the network.

The proposal model is a Mixed Integer Programming problem (MIP). The BARON solver was used, and the model was coded in the software GAMS.

3. <u>Mathematical Model</u>

The proposed model is based on the superstructure shown in Figure 2. The sets, variables and subscripts used in the model are defined in *Nomenclature* section.

Mass Balances

Water quality at the exit of each treatment plant and network consumption satisfy the quality constraints for their respective user. Stationary state is also assumed for each node. Finally, each flow density is assumed constant, then volumetric balances can be made.

Global Mass Balances

For the global mass balance it is assumed that all flows from surface and underground water sources are equal to the flows discharged into the environment plus the losses of each node that go into the sink node as shown in Equation 1.

$$\begin{gathered}\sum_w \sum_i F_{w\to i} = \sum_j \sum_k F_{j\to k} + \sum_h \sum_s F_{h\to s}, \forall\, w \in W, \\ \forall\, k \in K, \forall\, s \in S, \forall\, i \in \{CFW\}, \forall\, h \in \{WT\}, \forall\, j \in \{DNC\}\end{gathered} \tag{1}$$

In this equation, i corresponds to the water consuming nodes, j to those that discharge water into natural courses and h to all nodes which have water losses.

Mass Balances for each Node

At any node, the incoming flow rates will be the same as the outgoing flow rates, as a result of the steady state assumption. A general expression is shown in Equation 2.

$$\sum_{i\in ON_j} F_{i\to j} = \sum_{h\in DN_j} F_{j\to h} + \sum_{s\in S} F_{j\to s}\ \forall\, j \in \{AS_j\} \tag{2}$$

In this equation, j corresponds to each node in which mass balance is carried out in all sets except water source, discharge, or sink, i.e. AS_j; i represents the nodes of origin of the input flow rates, i.e., ON_j; and h represents the destination nodes of the output flow rates, i.e., DN_j.

Certain characteristics for consumption, treatment, distribution, and collection nodes are specified below.

a. Mass Balance for Consumption Nodes

Mass balance of residential and commercial consumption can be supplied with drinking quality. As all consumption and treatment nodes, there is a flow to the sink. This is expressed in Equation 3 and its representation is given by Figure 3.

$$\sum_{a_2 \in DS} F_{a_2 \to j} + \sum_{e_2 \in NS} F_{e_2 \to j} + \sum_{l_2 \in MA^s} F_{l_2 \to j} + \sum_{p_2 \in NA^s} F_{p_2 \to j} + \sum_{d \in D} F_{d \to j} = \sum_{c \in C} F_{j \to c} + \sum_{b_2 \in WS} F_{j \to b_2} + \sum_{p_2 \in NA^s} F_{j \to p_2} + \sum_{q_2 \in NB^s} F_{j \to q_2} + \sum_{r_2 \in NC^s} F_{j \to r_2} + \sum_{t_2 \in ND^s} F_{j \to t_2} + \sum_{l_2 \in MA^s} F_{j \to l_2} + \sum_{m_2 \in MB^s} F_{j \to m_2} + \sum_{n_2 \in MC^s} F_{j \to n_2} + \sum_{s \in S} F_{j \to s} \;\; \forall\, j \in \{RC \cup CC\} \tag{3}$$

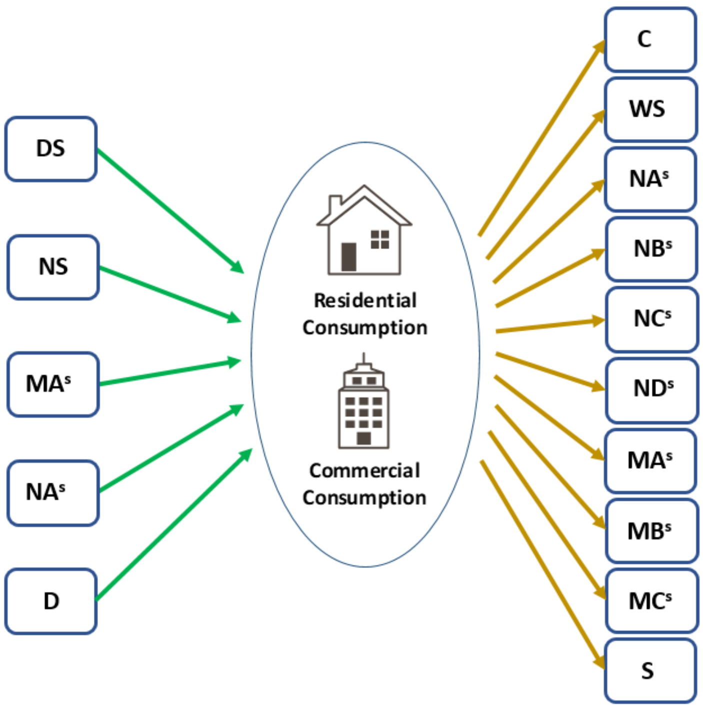


Figure 3: Representation of the mass balance of commercial and residential consumption.

Local industrial consumption can be supplied only with drinking water quality. Therefore, mass balance is determined by the Equation 4 and its representation is given by Figure 4.

$$\sum_{a_2 \in DS} F_{a_2 \to j} + \sum_{e_2 \in NS} F_{e_2 \to j} + \sum_{l_2 \in MA^s} F_{l_2 \to j} + \sum_{p_2 \in NA^s} F_{p_2 \to j} + \sum_{d \in D} F_{d \to j} = \sum_{g_5 \in EB} F_{j \to g_5} + \sum_{s \in S} F_{j \to s} \,, \;\; \forall\, j \in IB \tag{4}$$

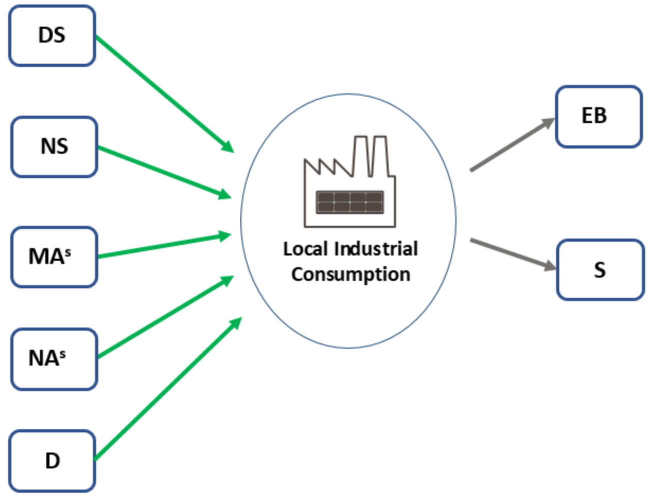


Figure 4: Representation of the mass balance of local industrial consumption.

Mass balances for large industrial consumers supplied with drinking water and discharging into surface water courses is presented in Equation 5 and its representation is shown in Figure 5. Mass balance for large industrial consumers supplied by fresh water and discharging to sewage system is presented in Equation 6 and its representation can be seen in Figure 6.

$$\sum_{d \in D} F_{d \to j} + \sum_{a_2 \in DS} F_{a_2 \to j} + \sum_{e_2 \in NS} F_{e_2 \to j} + \sum_{p_2 \in NA^s} F_{p_2 \to j} + \sum_{l_2 \in MA^s} F_{l_2 \to j} = \sum_{g_2 \in EL^{dn}} F_{j \to g_2} + \sum_{s \in S} F_{j \to s}\,, \ \forall\, j \in IL^{dn} \tag{5}$$

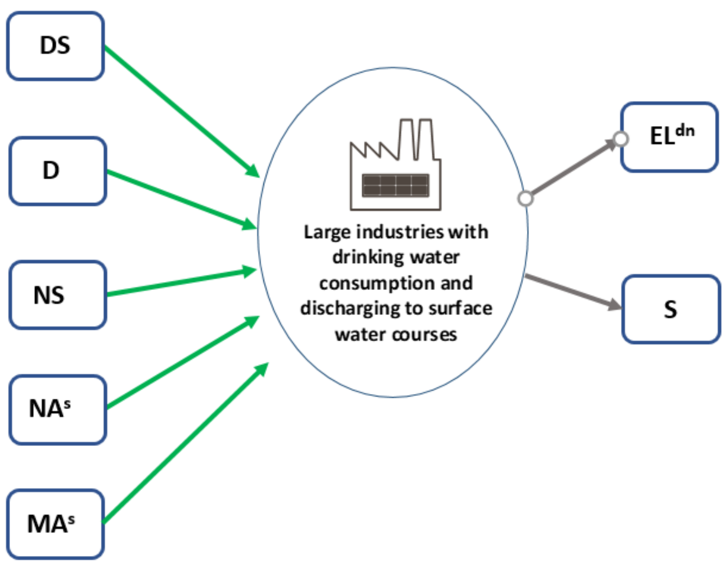


Figure 5: Representation of the mass balance of large industrial consumption supplied with drinking water and discharging into surface water courses.

$$\sum_{w \in W} F_{w \to j} + \sum_{q_1 \in NB^l} F_{q_1 \to j} + \sum_{q_2 \in NB^s} F_{q_2 \to j} + \sum_{m_1 \in MB^l} F_{m_1 \to j} + \sum_{m_2 \in MB^s} F_{m_2 \to j} = \sum_{g_3 \in EL^{fs}} F_{j \to s} + \sum_{s \in S} F_{j \to s}\,, \ \forall\, j \in IL^{fs} \tag{6}$$

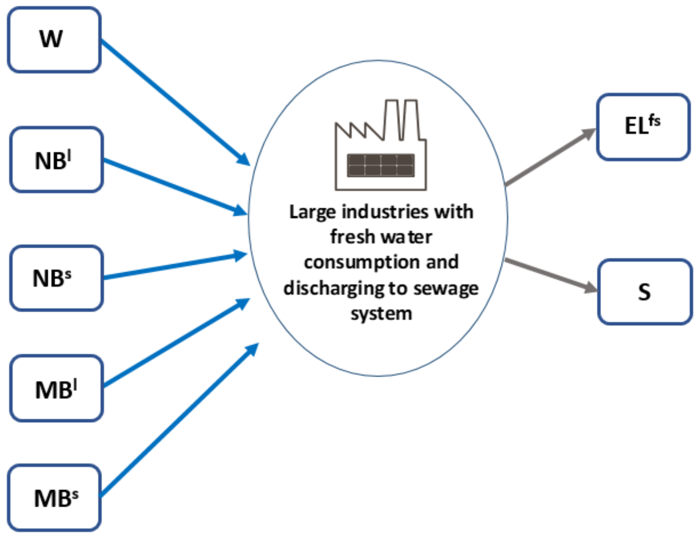


Figure 6: Representation of the mass balance of large industrial consumption supplied with fresh water and discharging into sewage system.

Mass balance of the agricultural irrigation consumption nodes is given by the Equation 7 and its representation is shown in Figure 7.

$$\sum_{w \in W} F_{w\to j} + \sum_{r_1 \in NC^l} F_{r_1\to j} + \sum_{r_2 \in NC^s} F_{r_2\to j} + \sum_{n_1 \in MC^l} F_{n_1\to j} + \sum_{n_2 \in MC^s} F_{n_2\to j} = \sum_{k_0 \in K^G} F_{j\to k_0} + \sum_{s \in S} F_{j\to s}\,,\ j \in AC \tag{7}$$

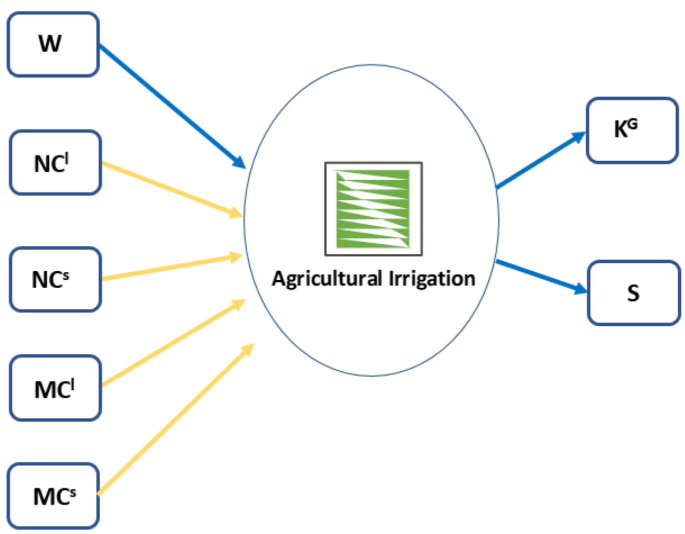


Figure 7: Representation of the mass balance of agricultural irrigation consumption.

Urban areas irrigation is subdivided in two subsets, depending on its source. The first one is supplied by drinking or fresh water; and the second, by fresh water. Mass balance of the first subset is given by the Equation 8 and its representation is given in Figure 8.

$$\sum_{w\in W} F_{w\to j} + \sum_{r_1\in NC^l} F_{r_1\to j} + \sum_{r_2\in NC^s} F_{r_2\to j} + \sum_{n_1\in MC^l} F_{n_1\to j} + \sum_{n_2\in MC^s} F_{n_2\to j} = \sum_{k_0\in K^G} F_{j\to k_0} + \sum_{s\in S} F_{j\to s}\,,\ \forall\, j \in PF \quad (8)$$

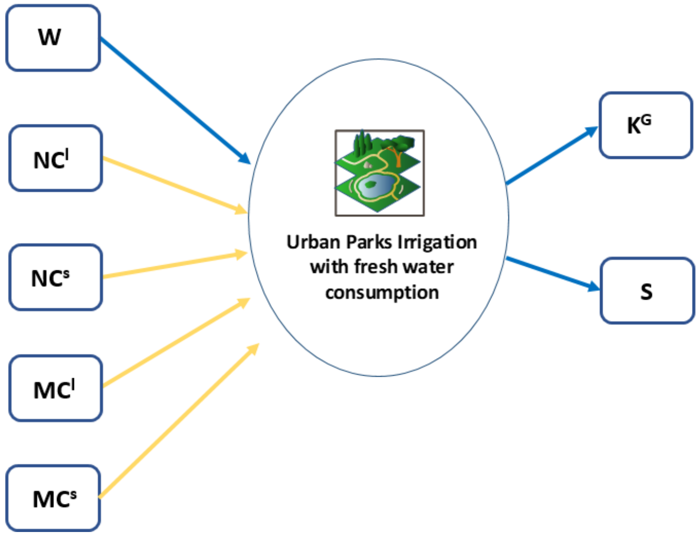


Figure 8: Representation of the mass balance of urban park irrigation with freshwater consumption.

Mass balance of the second subset is given by the Equation 9 and its representation is shown in Figure 9.

$$\sum_{a_2\in DS} F_{a_2\to j} + \sum_{d\in D} F_{d\to j} + \sum_{r_1\in NC^l} F_{r_1\to j} + \sum_{r_2\in NC^s} F_{r_2\to j} + \sum_{n_1\in MC^l} F_{n_1\to j} + \sum_{n_2\in MC^s} F_{n_2\to j} = \sum_{k_0\in K^G} F_{j\to k_0} + \sum_{s\in S} F_{j\to s}\,,\ \forall\, j \in PD \quad (9)$$

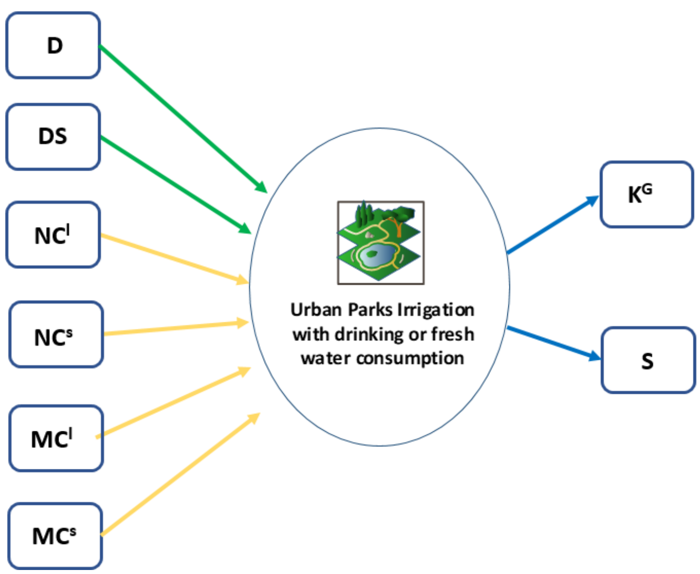


Figure 9: Representation of the mass balance of urban park irrigation with drinking or freshwater consumption

b. Mass Balances of Treatment Nodes

For existing treatment plants, mass balance of large existing DWTPs is determined by Equation 10 and its representation is given by Figure 10.

$$\sum_{w \in W} F_{w \to j} + \sum_{q_1 \in NB^l} F_{q_1 \to j} + \sum_{q_2 \in NB^s} F_{q_2 \to j} + \sum_{m_1 \in MB^l} F_{m_1 \to j} + \sum_{m_2 \in MB^s} F_{m_2 \to j} = \sum_{d \in D} F_{j \to d} + \sum_{s \in S} F_{j \to s}\,, \ \forall\, j \in DL \tag{10}$$

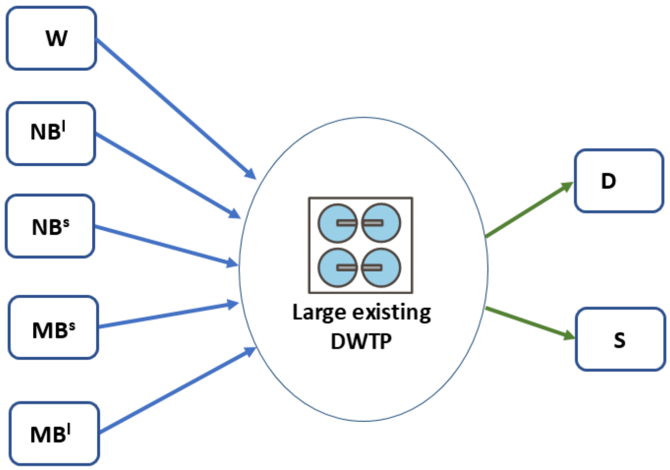


Figure 10: Representation of the mass balance of a large existing drinking water treatment plant.

Mass balance for small existing DWTPs is given by Equation 11 and its representation is shown in Figure 11.

$$\sum_{w \in W} F_{w \to j} + \sum_{q_1 \in NB^l} F_{q_1 \to j} + \sum_{q_2 \in NB^s} F_{q_2 \to j} + \sum_{m_1 \in MB^l} F_{m_1 \to j} + \sum_{m_2 \in MB^s} F_{m_2 \to j} = \sum_{i_1 \in RC} F_{j \to i_1} + \sum_{i_2 \in CC} F_{j \to i_2} + \sum_{u_1 \in PD} F_{j \to u_1} + \sum_{f_1 \in IL^{ds}} F_{j \to f_1} + \sum_{f_2 \in IL^{dn}} F_{j \to f_2} + \sum_{f_5 \in IB} F_{j \to f_5} + \sum_{s \in S} F_{j \to s}\,, \ \forall\, j \in DS \tag{11}$$

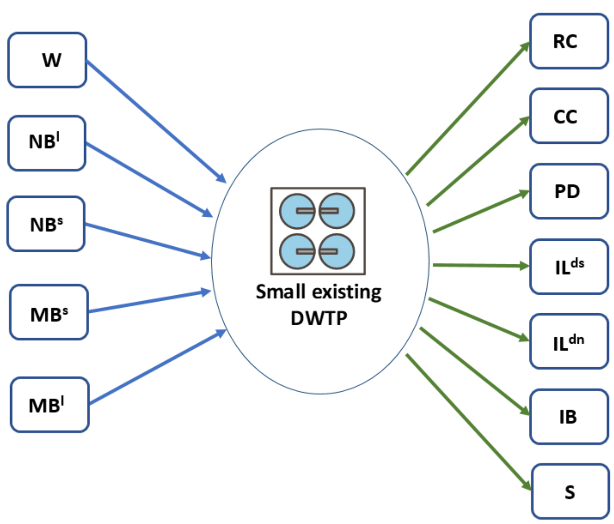


Figure 11: Representation of the mass balance for small existing drinking water treatment plant.

Mass balance of a large existing WWTP is given by Equation 12 and its representation is shown in Figure 12.

$$\sum_{c\in C} F_{c\to j} = \sum_{k_1\in K^s} F_{j\to k_1} + \sum_{s\in S} F_{j\to s}\,,\ \forall\, j \in WL \qquad (12)$$

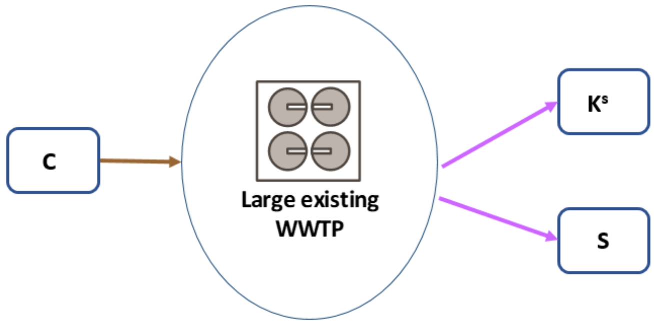


Figure 12: Representation of the mass balance of a large existing wastewater treatment plant.

Mass balance of small treatment plants is similar, but the incoming flows come directly from consumers: commercial, residential, and industrial consumers that discharge into the sewer system.

New DWTPs allow to install a new plant in a different location. Mass balances for new large and small DWTPs are the same as existing ones.

New WWTPs can achieve four new output qualities: (i) drinking water, (ii) fresh water, (iii) irrigation water, and (iv) discharge in natural course. Their respective mass balances are determined by Equations 13, 14, 15 y 16, respectively and their respective representations are given by Figure 13, 14, 15 and 16 for each case.

$$\sum_{c\in C} F_{c\to j} = \sum_{d\in D} F_{j\to d} + \sum_{s\in S} F_{j\to s}\,,\ \forall\, j \in NA^l \qquad (13)$$

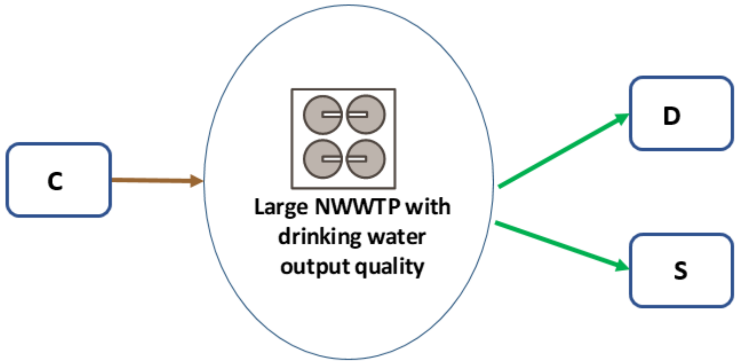


Figure 13: Representation of the mass balance of a large new wastewater treatment plant with drinking water output quality.

$$\sum_{c \in C} F_{c \to j} = \sum_{a_1 \in DL} F_{j \to a_1} + \sum_{a_2 \in DS} F_{j \to a_2} + \sum_{f_3 \in IL^{fs}} F_{j \to f_3} + \sum_{f_4 \in IL^{fn}} F_{j \to f_4} + \sum_{s \in S} F_{j \to s}, \ \forall\, j \in NB^l \tag{14}$$

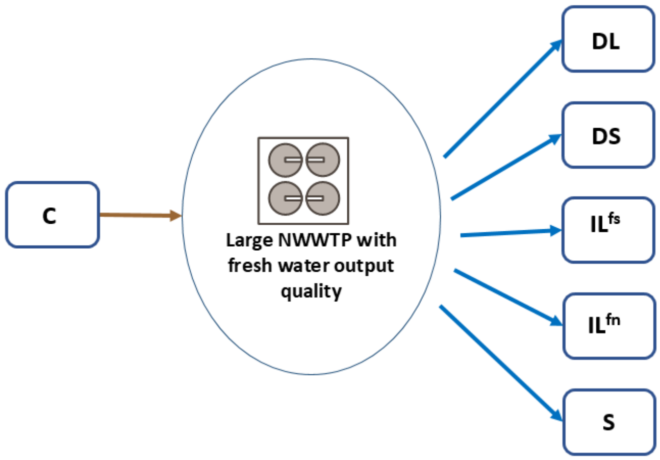


Figure 14: Representation of the mass balance of a large new wastewater treatment plant with fresh water output quality.

$$\sum_{c \in C} F_{c \to j} = \sum_{h_1 \in AC} F_{j \to h_1} + \sum_{u_1 \in PD} F_{j \to u_1} + \sum_{u_2 \in PF} F_{j \to u_2} + \sum_{s \in S} F_{j \to s}, \ \forall\, j \in NC^l \tag{15}$$

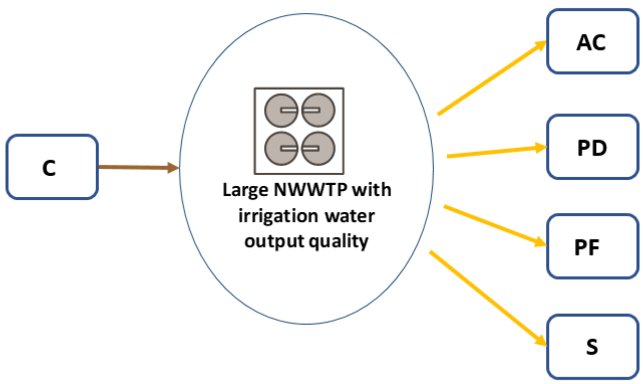


Figure 15: Representation of the mass balance of a large new wastewater treatment plant with irrigation water output quality.

$$\sum_{c \in C} F_{c \to j} = \sum_{k1 \in K^s} F_{j \to k1} + \sum_{s \in S} F_{j \to s}, \ \forall\, j \in ND^l \tag{16}$$

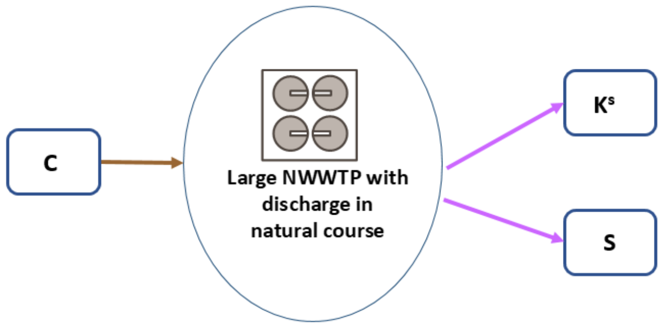


Figure 16: Representation of the mass balance of a large new wastewater treatment plant with discharge in natural course.

Modified wastewater treatment plants can be improved to achieve a different output quality: (i) drinking water, (ii) fresh water, and (iii) irrigation water. Their mass balances are the same as those of new WWTPs. For industrial wastewater treatment plants, the same logic as for their respective sources of consumption.

c. Mass Balances for Distribution and Collection Nodes

Distribution and collection nodes aim at representing the network of the water system. The mass balance of the distribution nodes is given by Equation 17 and its representation is shown in in Figure 17.

$$\sum_{a_1 \in DL} F_{a_1 \to j} + \sum_{e_1 \in NL} F_{e_1 \to j} + \sum_{p_1 \in NA^l} F_{p_1 \to j} + \sum_{l_1 \in MA^l} F_{l_1 \to j} = \sum_{i_1 \in RC} F_{j \to i_1} + \sum_{i_2 \in CC} F_{j \to i_2} + \sum_{u_1 \in PD} F_{j \to u_1} + \sum_{f_1 \in IL^{ds}} F_{j \to f_1} + \sum_{f_2 \in IL^{dn}} F_{j \to f_2} + \sum_{f_5 \in IB} F_{j \to f_5} + \sum_{s \in S} F_{j \to s}, \forall\, j \in D \tag{17}$$

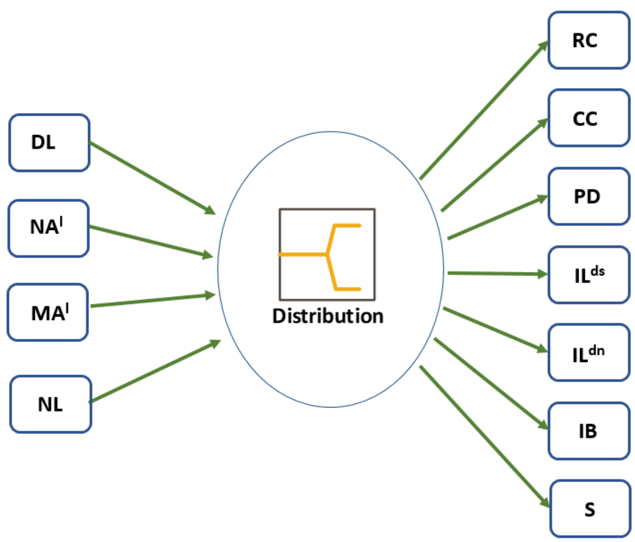


Figure 17: Representation of the mass balance of a distribution node.

Mass balance of collection nodes is determined by Equation 18 and its representation is given by Figure 18.

$$\sum_{i_1 \in RC} F_{i_1 \to j} + \sum_{i_2 \in CC} F_{i_2 \to j} + \sum_{g_1 \in EL^{ds}} F_{g_1 \to j} + \sum_{g_3 \in EL^{fs}} F_{g_3 \to j} + \sum_{g_5 \in EB} F_{g_5 \to j} = \sum_{b_1 \in WL} F_{j \to b_1} + \sum_{p_1 \in NA^l} F_{j \to p_1} + \sum_{q_1 \in NB^l} F_{j \to q_1} + \sum_{r_1 \in NC^l} F_{j \to r_1} + \sum_{t_1 \in ND^l} F_{j \to t_1} + \sum_{l_1 \in MA^l} F_{j \to l_1} + \sum_{m_1 \in MB^l} F_{j \to m_1} + \sum_{n_1 \in MC^l} F_{j \to n_1} + \sum_{s \in S} F_{j \to s}, \forall\, j \in C \qquad (18)$$

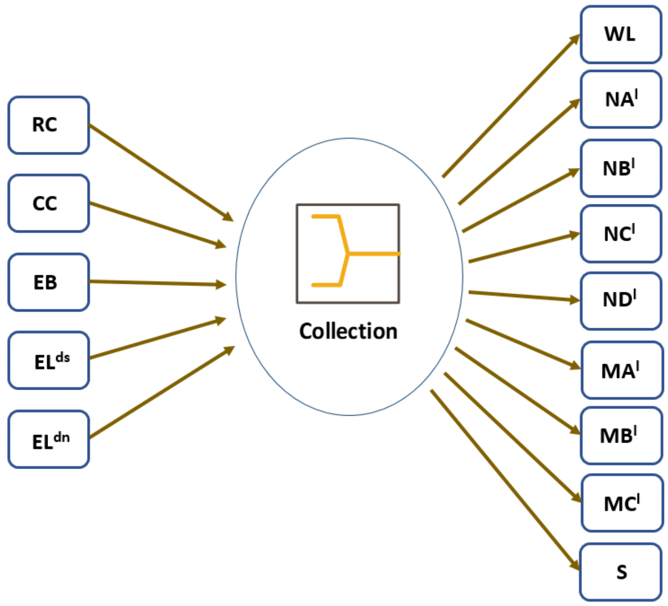


Figure 18: Representation of the mass balance of collection node.

Covering the Demand

To satisfy the water demand of each node, Equation 19 must be respected.

$$\sum_{i \in ON_j} F_{i \to j} \geq DM_{\{j,p\}}, \forall\, j \in AD_j, \forall\, p \in CT \qquad (19)$$

In this equation, i corresponds to nodes of origin of j, i.e. ON_j, j being the current node, i.e. where the demand must be satisfied (AD_j), and p represent each district.

Treatment Plant Capacity

In all capacity restrictions there is a territorial association of plants and consumers sharing the plant district, i.e. the capacity of a plant in the p' district, shall meet the demands of p' district users.

New Drinking Water Treatment Plant Capacity

As mentioned above, there are large and small treatment plants. The value separating both categories varies with the consumption of each district. In the case of big new drinking water treatment plants Equation 20 must be respected.

$$\sum_{d \in D} F_{i \to d} \geq \sum_j DM_{\{j,p'\}} \cdot E_i,\ \forall\, i \in NL,\ \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \qquad (20)$$

On the other hand, for small new drinking water treatment plants, Equation 21 must be respected. A new parameter '*m*', which corresponds to a small value in comparison to the demands of the district, is used to flexibilize the creation of small plants.

$$\sum_j F_{i \to j} \leq \left(\sum_j D\,M_{\{j,p'\}} + m\right) \cdot E_i,\ \forall i \in NS,\ \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \quad (21)$$

New Wastewater Treatment Plant Capacity

For new big wastewater treatment plants, Equation 22 must be respected.

$$\sum_c F_{c \to i} \geq \sum_j D\,M_{\{j,p'\}} \cdot E_i,\ \forall\, i \in \{NA^l \cup NB^l \cup NC^l \cup ND^l\},\ \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \quad (22)$$

For small new wastewater treatment plant, Equation 23 must be respected.

$$\sum_j F_{j \to i} \leq \left(\sum_j D\,M_{\{j,p'\}} + m\right) \cdot E_i,\ \forall i \in \{NA^s \cup NB^s \cup NC^s \cup ND^s\},\ \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \quad (23)$$

Existing or Modified Treatment Plant Capacity

Flow rates through existing or modified wastewater treatment plants must vary according to season. This fluctuation can be observed in Figure 19 and shows a higher consumption in summer than in winter. This logic is reflected in Equations 24 and 25. The percentages were obtained from the drinking water treatment plant in operation in Santiago.

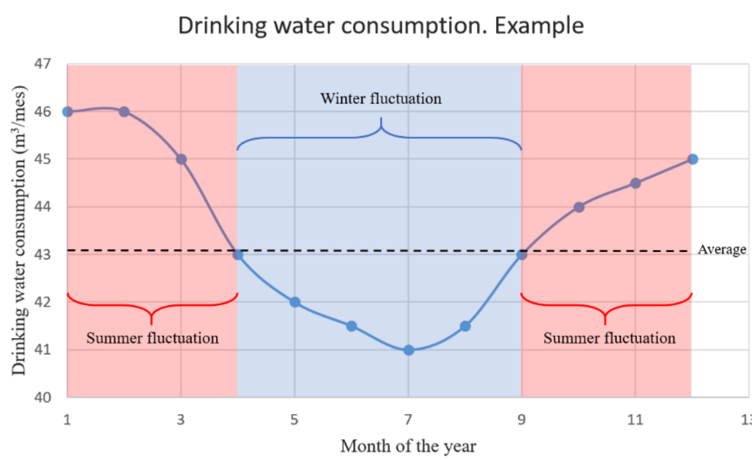


Figure 19: Drinking water consumption during a year. Illustrative example for the comprehension of the text. These values may vary each year.

$$\sum_{i \in ON_j} F_{i \to j} \geq 0.75 \cdot 0.85 \cdot AF_j \cdot E_j,\ \forall\, j \in EW \quad (24)$$

$$\sum_{i \in ON_j} F_{i \to j} \leq 1.23 \cdot AF_j \cdot E_j,\ \forall\, j \in EW \quad (25)$$

AF_j is the current flow rate treated by the plant, and E_j corresponds to the existence of the plant j.

In the case of drinking water treatment plants, the analysis is similar as shown in Equations 26 and 27.

$$\sum_{i \in DN_j} F_{j \to i} \geq 0.75 \cdot 0.85 \cdot AF_j \cdot E_j,\ \forall\, j \in ED \quad (26)$$

$$\sum_{i \in DN_j} F_{j \to i} \leq 1.23 \cdot AF_j \cdot E_j,\ \forall\, j \in ED \quad (27)$$

Logical Relationships of Existence

If the plant does not exist, then the incoming flows must be zero. This can be written mathematically trough Equation 28 applying the $BigM$ method [19].

$$\sum_i F_{i\to j} \leq M \cdot E_j \tag{28}$$

In this equation, i belongs to the set of origin nodes of the treatment plant j, i.e. ON_j, j corresponds to each wastewater treatment plant, M is a parameter, and E_j corresponds to the plant j existence.

In addition, there are also coexistence relations of plants: an existing and a modified plant cannot exist in the same place, as shown in the Equation 29.

$$\sum_{j\in\{EW\cup MW\}} E_j \leq 1 \tag{29}$$

Cost

The costs in the problem are divided in operational costs (*OpC*) and capital costs (*CapC*). OpC is given by the equation 30 and CapC is given by equation 31 [12]. The parameters used for the problem can be found in Appendix 1

$$OpC = T \cdot E \cdot \sum_j \sum_i F_{i\to j} \cdot \left(g \cdot (h_j - h_i) + f \cdot \frac{dist_{(i,j)} \cdot v^2}{2D}\right) \tag{30}$$

$$CapC = K_f \cdot E_i \cdot (\sum_i C\,vr_i \cdot \sum_j F_{j\to i} + Cfr_i + Ter_i) \tag{31}$$

Where 'OpC' is the operational cost of the system, 'T' is the annual operational time, 'E' is the cost per unit of electricity, 'f' is the Darcy factor, 'h_i' is the height of the node i, '$dist_{(i,j)}$' is the distance from node i to node j, 'v' is the linear velocity, 'g' is the gravity acceleration, 'D' is the diameter of each pipe, 'K_f' is the factor to annualize cost, , Cvr_i correspond to the investment cost parameter of plant i which is multiplied by the summation of the flows entering plant i, Cfr_i correspond to the intercept point in the ordinate, depending on the plant i, Ter_i correspond to the terrain cost of the plant I, and '$F_{i\to j}$' is the flow rate from node i to node j.

Objective Functions

The problem has two opposing objective functions to be minimized: water flow used from the water source (G0) and the total cost (TC). These two objective functions FO1 and FO2 can be represented by Equations 32 and 33, respectively.

$$FO1 = min \sum_{j\in CFW} F_{w\to j} = min\ \ G0,\ \forall\ w \in W \tag{32}$$

$$FO2 = min\,TC = min\,(OpC + CapC) \tag{33}$$

4. Multi-Objective Optimization Strategy

A goal programming method is applied to solve the MOO problem. The multi-objective formulation is shown in equation 34 subject to the constraints 35 and 36 :

$$min\ \gamma \tag{34}$$

$$\frac{G0-G0_{id}}{G0_{nid}-G0_{id}} \cdot w_1 \leq \gamma \qquad (35)$$

$$\frac{TC-TC_{id}}{TC_{nid}-TC_{id}} \cdot w_2 \leq \gamma \qquad (36)$$

The indices id and nid correspond to ideal and non-ideal solutions, respectively. Parameters w_1 and w_2 represent the relative weights of each objective function. γ represents the variable to be minimized.

5. Case Study

The model presented was applied in the city of Santiago, capital of Chile. Santiago is Chilean political, economic, and institutional center. The city has a population of six million people [20]. Santiago is located on the hydrographic basin of the Maipo River, where the main surface water sources are the Maipo and Mapocho Rivers. Santiago has been signed as a hydric stress zone, with an average regional deficit of 11.4% [21].

The system is represented by one large and one small drinking water treatment plant and one large and one small wastewater treatment plant. These 4 plants can treat all the real city flow. In the current system there is no water recycling, all wastewater treatment plants discharge their effluents into natural water courses.

A simplified version of the problem is presented. For simplicity, the city was divided into four areas as follows: North-East (NE or SL), North-West (NW or GF), South-East (SE or RC) and South-West (SW or HF). Population was distributed geographically according to the districts demographic information [20]. This division is shown in Figure 20. In addition, only one big industrial consumption, 2 wastewater treatment plants and 2 drinking water treatment plants were considered. Consumption is the same as in real problem, since the districts were clustered only in fewer points, but with the same water use. The flow of the entire network is representative.

For characteristics of the different districts, including consumption, locations, costs, and losses, see Appendix 1.

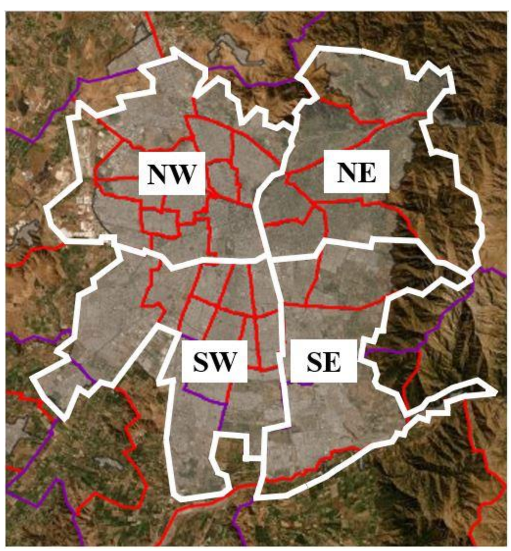


Figure 20: Division of Santiago for the simplified version. Modified image from INE [22].

6. Results

The MIP model of the case study has 484 constrains, 1,075 variables (including 50 binary variables), and was executed in an INTEL CORE i7-7700 HQ computer with 16 GB of memory. After a CPU time of 0.125 seconds the result was 16.06 $[m^3/s]$ of flow extracted from the water source and a total cost of 945,932,577[USD/year], when the weights of both functions are equal.

With the results of Table 2, the multiobjective problem is formulated obtaining the Pareto curve shown in Figure 22, where the values at the extremes of the curve were removed to make other intermediate values more clearly visible. The complete Pareto curve is shown in Figure 21.

Table 2: Results for mono-objective problems

Variable in objective function	Ideal value	Nonideal value
Flow rate from water source: G0 (m^3/s)	14.67	25.59
Total Cost: TC (USD/year)	940,797,564	980,974,542

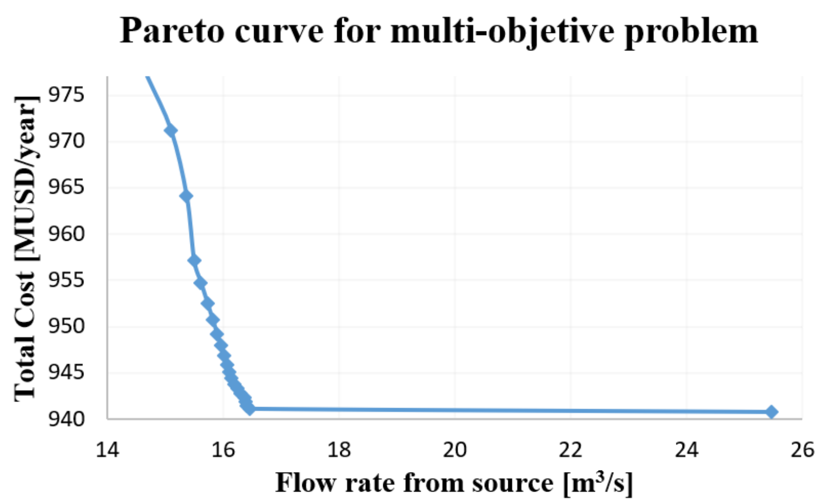


Figure 21: Pareto curve obtained for the multi-objective problem.

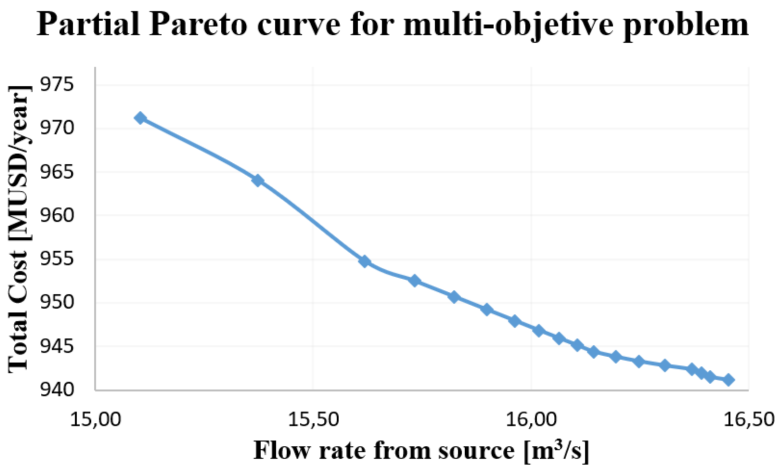
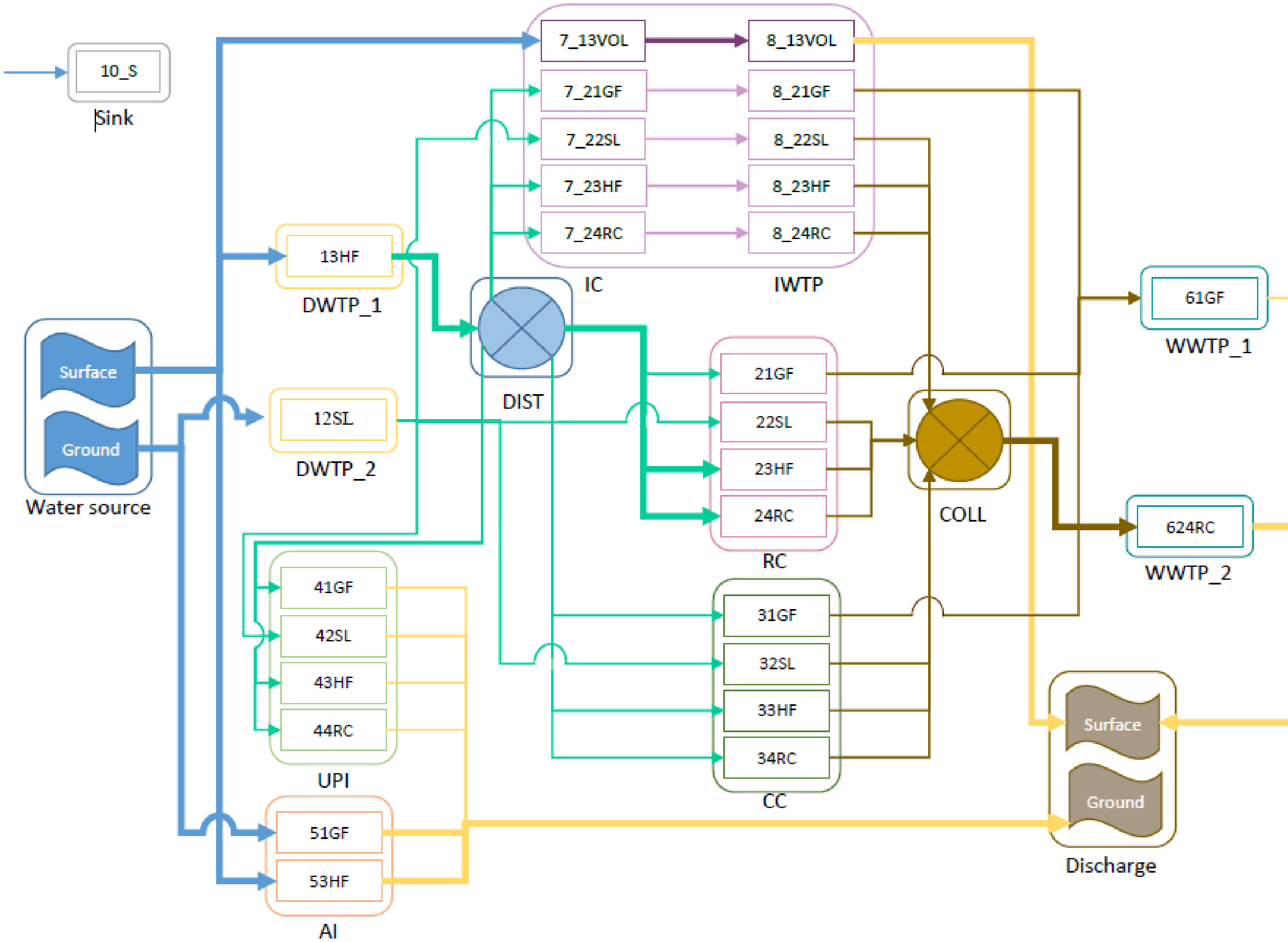


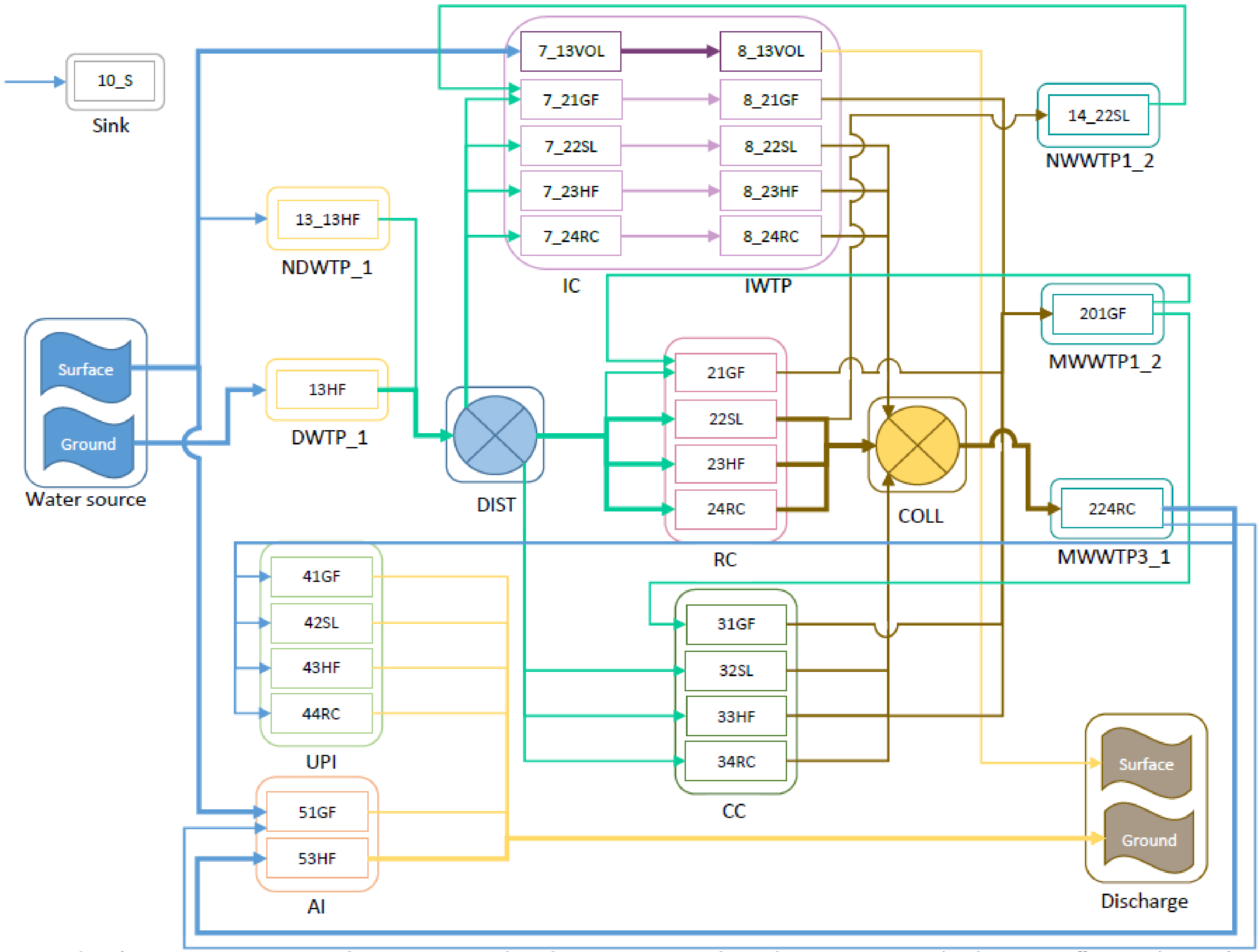
Figure 22: Section of the Pareto curve obtained for the multi-objective problem.

For equal importance in both objective functions, i.e, w_1=w_2=0.5 in equations 35 and 36, the superstructure shown in Figure 24 is obtained. The mass balance for the complete system is shown in Figure 26. This can be compared to the current base case represented by Figure 23, where the overall mass balance is shown in Figure 25. The optimized configuration reduces the water extraction from the natural source in 35.7%. In this solution, the annualized total cost of the water network grows a 0.5% when compared with the solution at the economic extreme of the Pareto curve, when the importance of the economic objective function is complete.



RC: Residential Consumption; **CC**: Commercial Consumption; **IC**: Industrial Consumption; **UPI**: Urban Parks Irrigation; **AI**: Agricultural Irrigation. Different qualities can be observed: in light blue, source quality; in green, drinking water quality; in brown, sewer discharge quality; and in yellow, surface or underground discharge quality.

Figure 23: Superstructure for the current situation of the city.



RC: Residential Consumption; **CC**: Commercial Consumption; **IC**: Industrial Consumption; **UPI**: Urban Parks Irrigation; **AI**: Agricultural Irrigation. Different qualities can be observed: in light blue, source quality; in green, drinking water quality; in brown, sewer discharge quality; and in yellow, surface or underground discharge quality.

Figure 24: Superstructure for the optimized solution with equal weights for both objective functions.

Figure 25: Global mass balance for the current situation of the city.

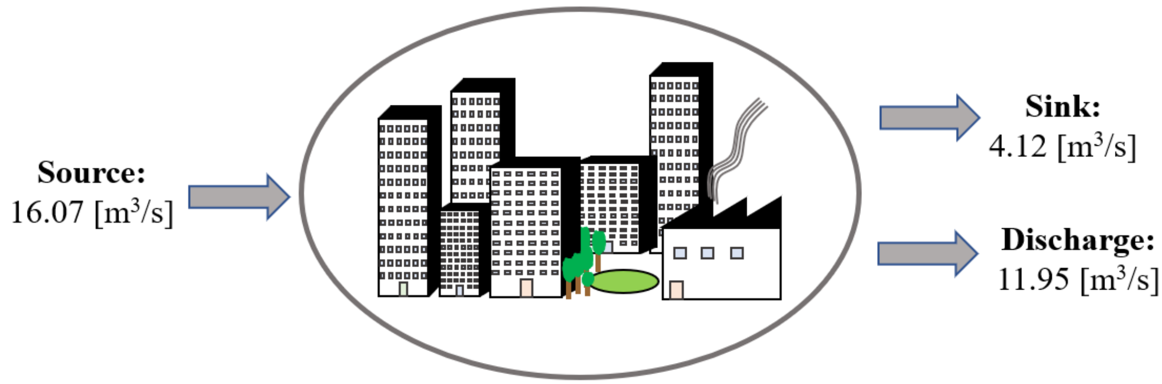


Figure 26: Global mass balance for the optimized problem with equal weights in both objective functions.

If no new or modified plants are imposed on the model, then the solution is the point where $w_1 = 0$ and $w_2 = 1$ on the Pareto curve, i.e. the last point on the curve with the highest value of $G0$ and the lowest value of TC. This partially represents the current situation, since, while the treated flow is the same as when the environmental indicator is not important, the cost is higher since flows are reconfigured within the network in a suboptimal way.

In summary, Table 3 shows a comparison between current system and optimized system.

Table 3: Results for multi-objective problem

Plant type	Current system	Optimized system
DWTP	2	2
WWTP with discharge quality output	2	0
WWTP with potable quality output	0	2
WWTP with irrigation quality output	0	1

7. Discussions

With respect to the obtained results, the current network is not optimal for water treatment, under the assumption that both objectives have the same importance. The optimal network considers the installation of a new drinking water treatment plant and a new wastewater treatment plant. It is also observed the presence of large and small plants in the case of drinking and wastewater, using the established distribution and collection networks, respectively. Both existing wastewater treatment plants are modified, and a new small plant is installed. This is mainly due to the high costs of installing new plants, compared with the modification of existing ones. The modification of plants is realistic, since installation costs are high when compared with operation costs, because of land costs. This behavior approaches other international cases such as Singapore, where there are treatment plants on the roof of other existing plants [24].

The result includes small treatment plants. According to the model, these plants distribute or collect directly from their consumers. This is a simplification, as they do not connect to a particular type of consumer. However, these plants connect to a small district network, which in turn connects to individual consumers. For example, commercial and residential consumption, are found in all districts (GF, HL, SL, RC), however it is shown in Figure 24 that the modified WWTP with drinking quality output ($MWWTP1\_2$) feeds the GF district consumption, while the distribution network feeds the remaining ones. Thus, the modified WWTP sends two flow rates, one towards commercial GF consumption and the other towards residential GF consumption: in order to implement, these flows are a single flow rate that is divided in the different consumption points, since they are in the same district.

The optimal result includes the installation of a new small DWTP instead of a new large DWTP, in order to supply drinking water consumers.

The water extracted is mainly used for irrigation and large industrial consumption, since there are no costs associated with transport, nor losses from water treatment. As there are other consumers to supply, WWTP needs to be installed or modified. There are two options: fresh water or drinking water output quality. With the first option, the flow must be connected to a DWTP plant for water treatment, so the costs and losses are higher. On the other hand, the second option allows large or small plants. The installation and modification of small WWTP reduces water losses associated with distribution and collection nodes.

Existing wastewater treatment plant was modified to supply irrigation water to different consumers. This quality allows to reduce water losses, compared to freshwater quality (which is connected to a DWTP, generating a extra loss for its treatment), or to the discharge quality (which is sent directly to natural courses).

Another interesting result is that no decision is made to install wastewater treatment plants with discharge output quality. This is because it would incur in high installation costs without being able to recycle the water. In addition, no decision is made to install wastewater treatment plants with natural source output quality either. This is interesting because in many countries, water is injected into the water table to refill it with naturally filtered water [23]. Besides our model shows this solution as non-optimal in environmental and economic aspects, the advantage of this type of plant is social preference, as they make an indirect reuse of the water in the network [24].

The Pareto curve shows the solutions for objective functions when they have different combinations of relative importance. A trade-off of both functions is observed, where the relation between both functions is not direct. Moving through the Pareto curve, plants are installed or uninstalled, and this behavior is not linear. This causes some important differences in the costs of a point compared to its previous or subsequent point, visually this is observed as changes in the gradients of each point.

As a projection, it may be important to include the choice of different types of technologies to be installed as an optimization variable, not as a given parameter according to quality. Another combination of technologies could be economically or environmentally better depending on the regional context. It is also important to improve the districts configuration. This improvement can include a mapping of pipes in order to refine transport cost estimation within the districts.

Finally, it is planned to incorporate the limits of extraction flow through an analysis of the effects of Climate Change on different variables in Santiago. As mentioned before, the consumption of water can vary through years because of demographic pressure and Climate Change, and the sources of water could also change because of the same reason.

Conclusions

This paper deals with the management of water resources by integrating new water treatment plants to find the optimal configuration of the water network, applied to the case study of Santiago, Chile.

The characterization of the regional water use is addressed dividing the participants within the network in sets and defining parameters as demand, consumption, losses, location, and costs. This organized structure is an important framework to look for an improved configuration of the regional water network with a formal optimization technique.

An MIP was formulated with this purpose, where the discontinuous variables were given by the existence of drinking water treatment plants and wastewater treatment plants with different output qualities, to allow water recycling within the network. This decision of allowing different qualities for water within a system is important to assess and compare the significant number of possible recycling and reuse connections within a city.

The problem seeks to minimize the costs of installing plants and of transporting water as well as to minimize the flow extracted from the natural source.

To apply the methodology to Santiago, it was necessary to separate the city into four districts in order to establish the characteristics of the different nodes required. Pareto curve was found with optimal solutions for different weights of each objective function. In the case in which both functions have the same importance, some ideas can be listed as an heuristic to compare possible water networks in other cities: (i) it was found that the optimal solution involves installing a new drinking water treatment plant and a new wastewater treatment plant with drinking output qualities. This solution also proposes (ii) the modification of the two existing wastewater treatment plants to reach drinking and irrigation quality outputs respectively. These results allow a reuse of water within the network, where the modification of the current WWTPs is preferred, because of the high cost of installation of new plants. For plants with drinking water quality, (iii) the use of small plants is preferred since the losses generated by distribution/collection are reduced. These results show that (iv) it is more environmentally and economically convenient to reuse water for irrigation and drinking consumption rather than recycling water to the natural source.

Santiago can reduce the water extraction from the natural source in 35.7%. In this optimal solution, the annualized total cost of the water network grows a 0.5% when compared with the economic extreme of the Pareto curve.

Finally, the model can be implemented in other contexts, allowing better planning of water resource in any city. The proposed model is a formal strategy to help decision-makers to improve water resource planning for any city with simultaneous environmental and economic objectives.

Acknowledgments

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Nomenclature

Parameters

DM_k	Water demand of node k, see table 4
AF_i	Current flowrate of existing plant i, see table 5
M	Big M, 1000000
m	Little m, 0.02
K_f	Factor to annualize the capital cost, 1/20 ($year^{-}1$)
T	Annual operating time [h-year], 8760
E	Electricity cost [USD-kWh], 0.158
f	Darcy coefficient [], 0.009
g	Gravity acceleration [m-s^2], 9.8
v	Average velocity [m-s], 1.5

Variables

$F_{i,j}$	Water flow from node i to node j
E_i	Existence of node i
γ	Variable
w_i	Weight

Sets

W	Set for water sources
ED	Existing Drinking Water Treatment Plants
EW	Existing Waste-Water Treatment Plants
IE	Industrial Effluent Treatment Plants
$ND0$	New Drinking Water Treatment Plants
MW	Modified Waste-Water Treatment plants
NW	New Waste-Water Treatment Plant
RC	Residential Consumption
CC	Commercial Consumption
PC	Urban Park Irrigation
AC	Agricultural Irrigation
IC	Industrial Consumption
D	Set for Distribution nodes
C	Set for Collection nodes
K	Set for discharge natural courses
S	Sink

Subsets

W^G	Groundwater sources
W^S	Surface water sources

DL Large Existing DWTPs
DS Small Existing DWTPs
WL Large Existing WWTPs
WS Small Existing WWTPs
PD Urban Park Irrigation with drinking water consumption
PF Urban Park Irrigation with fresh water consumption
IL Large Industrial consumers
IB By-district Industrial consumers
EL Industrial Effluent Treatment Plant connected with large industrial consumers
EB Industrial Effluent Treatment Plant connected with by-district industrial consumers
NL New Large DWTPs
NS New Small DWTPs
MA Modified WWTPs with drinking water output quality
MB Modified WWTPs with fresh water output quality
MC Modified WWTPs with irrigation water output quality
NA New WWTPs with drinking water output quality
NB New WWTPs with fresh water output quality
NC New WWTPs with irrigation water output quality
ND New WWTPs with discharge water output quality
K^G Natural groundwater discharge courses
K^S Natural surface discharge courses

Subsubsets

DG Existing Small DWTPs with groundwater consumption
DS Existing Small DWTPs with surface water consumption
IL^d Large Industrial consumers with drinking water consumption
IL^f Large Industrial consumers with fresh water consumption
EL^d Related to IL^d
EL^f Related to IL^f
MA^l Modified Large WWTPs with drinking water output quality
MB^l Modified Large WWTPs with fresh water output quality
MC^l Modified Large WWTPs with irrigation water output quality
MA^s Modified Small WWTPs with drinking water output quality
MB^s Modified Small WWTPs with fresh water output quality
MC^s Modified Small WWTPs with irrigation water output quality
NA^l New Large WWTPs with drinking water output quality
NB^l New Large WWTPs with fresh water output quality
NC^l New Large WWTPs with irrigation water output quality
ND^l New Large WWTPs with discharge water output quality

NA^s New Small WWTPs with drinking water output quality

NB^s New Small WWTPs with fresh water output quality

NC^s New Small WWTPs with irrigation water output quality

ND^s New Small WWTPs with discharge water output quality

Subsubsubsets

IL^{ds} Large Industrial consumers with drinking water consumption and sewage discharge

IL^{dn} Large Industrial consumers with drinking water consumption and natural water course discharge

IL^{fs} Large Industrial consumers with fresh water consumption and sewage discharge

IL^{fn} Large Industrial consumers with fresh water consumption and natural water course discharge

EL^{ds} Related to IL^{ds}

EL^{dn} Related to IL^{dn}

EL^{fs} Related to IL^{fs}

EL^{fn} Related to IL^{fn}

Other types of sets not included in the formulation

ON_j Origin nodes for each mass balance in the node $'j'$

DN_j Destination nodes for each mass balance in the node $'j'$

AS_j All set except for water sources, discharge natural courses or sink. Is the node $'j'$

CFW Sets consuming fresh water, i.e. $\{ED \cup ND0 \cup PC \cup AC \cup IC\}$

WT Sets that have water losses, i.e. $\{ED \cup EW \cup IE \cup ND0 \cup MW \cup NW \cup RC \cup CC \cup PC \cup AC \cup IC \cup D \cup C\}$

DNC Sets that discharge water in natural courses, i.e., $\{EW \cup EL^{dn} \cup EL^{fn} \cup ND\}$

AD_j Set that includes the consumption nodes,i.e., $\{RC \cup CC \cup PC \cup AC \cup IC\}$

CT Set representing each district

General subscripts

w Water source node related to set W

d Distribution node related to set D

c Collection node related to set C

k Discharge natural course node related to set K

i_1 Residential consumption node related to set RC

i_2 Commercial consumption node related to set CC

h_1 Agricultural irrigation node related to set AC

u_1 Urban Park Irrigation node with drinking water consumption related to subset PD

u_2 Urban Park Irrigation node with fresh water consumption related to subset PF

f_1 Large Industrial consumption node with drinking water consumption and sewage discharge related to subsubsubset IL^{ds}

f_2 Large Industrial consumption node with drinking water consumption and natural water course discharge related to subsubsubset IL^{dn}

f_3 Large Industrial consumption node with fresh water consumption and sewage discharge related to subsubsubset IL^{fs}

f_4 Large Industrial consumption node with fresh water consumption and natural water course discharge IL^{fn}

f_5 By-district Industrial consumption node related to IB

k_0 Natural groundwater discharge courses related to subset K^G

k_1 Natural surface discharge courses related to subset K^S

Treatment plants subscripts

a_1 Large existing DWTP node related to subset DL

a_2 Small existing DWTP node related to subset DS

b_1 Large existing WWTP node related to subset WL

b_2 Small existing WWTP node related to subset WS

g_1 Industrial Effluent Treatment Plant node related to subsubsubset EL^{ds}

g_2 Industrial Effluent Treatment Plant node related to subsubsubset EL^{dn}

g_3 Industrial Effluent Treatment Plant node related to subsubsubset EL^{fs}

g_4 Industrial Effluent Treatment Plant node related to subsubsubset EL^{fn}

g_5 Industrial Effluent Treatment Plant node related to subset EB

e_1 New Large DWTP node related with subset NL

e_2 New Small DWTP node related with subset NS

l_1 Modified Large WWTP node with drinking water output quality related to subsubset MA^l

l_2 Modified Small WWTP node with drinking water output quality related to subsubset MA^s

m_1 Modified Large WWTP node with fresh water output quality related to subsubset MB^l

m_2 Modified Small WWTP node with fresh water output quality related to subsubset MB^s

n_1 Modified Large WWTP node with irrigation water output quality related to subsubset MC^l

n_2 Modified Large WWTP node with irrigation water output quality related to subsubset MC^s

p_1 New Large WWTP node with drinking water output quality related to subsubset NA^l

p_2 New Small WWTP node with drinking water output quality related to subsubset NA^s

q_1 New Large WWTP node with fresh water output quality related to subsubset NB^l

q_2 New Small WWTP node with fresh water output quality related to subsubset NB^s

r_1 New Large WWTP node with irrigation water output quality related to subsubset NC^l

r_2 New Small WWTP node with irrigation water output quality related to subsubset NC^s

t_1 New Large WWTP node with discharge water output quality related to subsubset ND^l

t_2 New Small WWTP node with discharge water output quality related to subsubset ND^s

Appendix 1: Parameters for the case study

The consumption of the different districts are shown in Table 4, where the large industrial consumption is 1.18 m3/s, located in the southeastern district (SE). The treatment flows of the current treatment plants are shown in Table 5. In addition, factors related to water losses at each node are shown in Table 6. The locations of the elements are in Table 7. The 4 average pipe diameters are shown in Table 8, the Cvr_i parameter for the installation cost is shown in Table 9 and the Ter_i parameter for the terrain cost, is given by Table 10.

Table 4: Consumption of the different districts (DM_k)

District	Residential demand (m^3/s)	Commercial demand (m^3/s)	Urban park irrigation (m^3/s)	Agricultural irrigation (m^3/s)	Industrial demand (m^3/s)
NE	0.98	0.32	0.23	2.00	0.09
NW	3.52	0.31	0.27	0	0.38
SE	2.69	0.39	0.26	5.91	0.13
SW	2.76	0.42	0.31	0	0.14

Table 5: Current flowrate of the treatment plants (AF_i)

Treatment plant	Present flowrate (m^3/s)
Big drinking water treatment plant (located in SE)	10.53
Small drinking water treatment plant (located in NW)	5.38
Big waste water treatment plant (located in NE)	7.76
Small waste water treatment plant (located in SW)	1.23

Table 6: Water loss factors in each set.

Set	Water loss factor
Big drinking water treatment plant	0.05
Small drinking water treatment plant	0.2
Big wastewater treatment plant	0.05
Small wastewater treatment plant	0.2
Industrial wastewater treatment plant	0.05
Residential consumption	0.1
Commercial consumption	0.1
Urban park irrigation	0.55
Agricultural irrigation	0.66
Industrial consumption	0.15
Distribution	0.15

Collection	0.15

Table 7: Locations of different districts, considering the lower left corner as the origin of the plan.

Set	Element	Horizontal Position X [km]	Vertical Position Y [km]	Height [m]
W	Water source in SE	124	0	787
ED	Treatment plant in SE	113.6	10.3	649
ED	Treatment plant in SE	113.6	10.3	649
ED	Treatment plant in NW	20.6	103.3	501
RC	Consumption in NE	93	93	683
RC	Consumption in NW	31	93	515
RC	Consumption in SE	93	31	616
RC	Consumption in SW	31	31	507
CC	Consumption in NE	93	93	683
CC	Consumption in NW	31	93	515
CC	Consumption in SE	93	31	616
CC	Consumption in SW	31	31	507
PC	Consumption in NE	93	93	683
PC	Consumption in NW	31	93	515
PC	Consumption in SE	93	31	616
PC	Consumption in SW	31	31	507
AC	Consumption in NE	93	93	683
AC	Consumption in SE	93	31	616
EW	Treatment plant in SW	10.3	31	504
EW	Treatment plant in NE	113.6	82.6	653
IC	Big consumption in SE	72.3	20.6	634
IC	Consumption in NE	93	93	683
IC	Consumption in NW	31	93	515
IC	Consumption in SE	93	31	616
IC	Consumption in SW	31	31	507
IE	Big treatment plant in SE	72.3	20.6	634
IE	Treatment plant in NE	93	93	683
IE	Treatment plant in NW	31	93	515
IE	Treatment plant in SE	93	31	616
IE	Treatment plant in SW	31	31	507
D	Distribution	62	62	569
C	Collection	62	62	569
ND	New treatment plant in NE	124	124	1000

ND	New treatment plant in NW	0	124	490
ND	New treatment plant in SE	124	0	790
ND	New treatment plant in SW	0	0	420
NW	New treatment plant in NE	124	124	1000
NW	New treatment plant in NW	0	124	490
NW	New treatment plant in SE	124	0	790
NW	New treatment plant in SW	0	0	420
MW	Modified treatment plant in SW	10.3	31	504
MW	Modified treatment plant in NE	113.6	82.6	653

Table 8: Average diameters for transport cost

Diameter 1 [m]	1.8
Diameter 2 [m]	1.2
Diameter 3 [m]	1
Diameter 4 [m]	0.5

Table 9: Cvr_i and Cfr_i for each treatment plant

Subset	$Cvr_i \left[\frac{MUSD \cdot s}{m^3}\right]$	Cfr_i $[MUSD]$
NA^l	2.19	54.83
NB^l	1.20	44.69
NC^l	1.04	34.51
ND^l	1.03	34.09
NA^s	3.65	3.87
NB^s	2.32	3.87
NC^s	1.76	2.18
ND^s	1.69	2.01
MA^l	1.99	1.78
MB^l	1.51	1.16
MC^l	0.02	0.06
MA^s	2.22	2.18
MB^s	0.61	1.78
MC^s	0.03	0.02

NL	0.07	0.59
NS	0.05	0.63

Table 10: Ter_i for each treatment plant

Subsets or subsubsets	District	Ter_i $[MUSD/year]$
NA^l, NB^l, NC^l, ND^l	GF	17.15
	SL	11.46
	HF	4.61
	RC	5.28
NA^s, NB^s, NC^s, ND^s	GF	4.29
	SL	2.87
	HF	1.15
	RC	1.32
NL	GF	5.72
	SL	3.82
	HF	1.53
	RC	1.76
NS	GF	1.43
	SL	0.95
	HF	0.38
	RC	0.44