A Multi-Objective Optimization Model to Plan City-Scale Water Systems with Economic and Environmental Objectives: A Case Study in Santiago, Chile

Daniela Gormaz-Cuevas, Bachelor Student, Departamento de Ingeniería Química, Biotecnología y Materiales, Universidad de Chile.

Javiera Riffo-Rivas, Project Engineer, Departamento de Ingeniería Química, Biotecnología y Materiales, Universidad de Chile.

Ludovic Montastruc, Associate Professor, Laboratoire de Génie Chimique, Université de Toulouse.

Felipe Díaz-Alvarado, Assistant Professor, Departamento de Ingeniería Química, Biotecnología y Materiales, Universidad de Chile (corresponding author)

felidiaz@ing.uchile.cl
(+56) 9 6218 3843
Av. Beauchef 851, Piso 6-poniente, Group of Sustainable Design and Process Systems Engineering, 8370456 Santiago, Chile.

Abstract

Climate Change and its effects in water scarcity has become an important challenge for cities with water management problems. These problems require an integral planning of the city, which can be supported by optimization. The main goal of the research is to provide a regional optimization model for water networks, including new treatment options. The model is formulated as a multi-objective mixed-integer programming problem, focused on environmental and economic impact of the network, minimizing water extracted from natural sources and total cost. The formulation is developed with the goal-programming methodology. The model covers a complete existing city-scale water network, including 4 different options of water reuse within the city: drinking water, fresh water, irrigation, and discharge in natural courses. The case study is Santiago, capital of Chile, which is the political, economic, and institutional center of Chile. If both objective functions have equal importance to configure the solution, the following ideas characterize the optimal water network: (i) it is more environmentally and economically convenient to reuse water within the network rather than recycling water to the natural source; (ii) the reuse of water is preferred in the form of irrigation and drinking qualities rather than industrial qualities to reduce transport costs, and (iii) the modification of the current treatment plants is preferred, because of the high cost of installation of new plants. An environmental and cost-effective solution for Santiago, Chile, can reduce the source water extraction in 35.7%. The model can be implemented in other contexts, providing orientations to decision-makers to plan city-scale water networks with simultaneous environmental and economic considerations.

Keywords: Water Networks, Regional Integration, Multi-Objective Optimization
1. Introduction

Climate change has become an important issue in several regions worldwide since human behavior is conditioned by the effects of Global Warming on Earth \[1\]. A direct effect is water scarcity \[2\], considered a global risk by the World Economic Forum \[3\]. To address this issue, long term planning of city-scale water systems has become a key matter. In this context, optimization techniques can be a valuable tool for water resource management in order to redesign regional water systems. This topic constitutes the focus of this project.

Some efforts have previously been made to model and optimize both Industrial Water Networks (IWN) and water networks within Eco-Industrial Parks (EIP), having environmental and economic benefits.

Campos de Faria et al. (2009) looked for the minimum operational cost and freshwater consumption of an IWN \[4\]. The results showed that it is useful to identify reuse opportunities. Boix et al. (2012) developed a multi-objective optimization model of an EIP \[5\]. Liu et al (2011) developed an optimization model for water resources management for insular areas in Greece \[6\]. Rojas-Torres et al. (2015) proposed a multi-objective optimization model to design a water system in a city scale, using water reuse for agricultural purpose \[7\]. Pérez et al. (2017) also proposed a model to design a water system in a city in Mexico but using rainwater reservoirs \[8\].

The objective when redesigning water systems is to sustainably improve the use of the resource. With this purpose, incorporating non-conventional water sources and the recycling wastewater have an important potential. Many efforts have been made to reuse and recycle water.

Lovelady and El-Halwagi (2009) developed a model to plan water management among multiple processes in a EIP facility \[9\]. Campos de Faria et al. (2009) proposed alternatives to optimize IWN using different regeneration units \[4\]. Sadegh et al. (2011) presented a model to minimize the energy of an inter-plant water network in an EIP \[10\].

Some studies refer to the incorporation of new sources in a city-scale optimization model. Liu et al (2011) presented an optimization approach for water management of a city including desalinated seawater and reclaimed water as water sources \[6\]. Rojas-Torres et al. (2015) incorporated rainwater harvesting and reclaimed water \[7\].

Other option is to retrofit existing water networks in order to redesign them. Campos de Faria et al. (2009) presented a methodology for retrofitting an IWN \[4\]. Sotelo-Pichardo et al. (2011) proposed a mathematical programming model for the optimal retrofitting of an IWN \[11\]. Rubio-Castro et al. (2012) developed a model to design an EIP by retrofitting existing water networks \[12\].

As cited above, research with plant modification is mainly applied in IWN. The present paper includes the possibility of installing new plants or retrofitting existing ones on a city scale.

Water system modification has different impacts. These impacts have been studied considering mainly economic or environmental dimensions.

Some authors focused on reducing the associated cost. Bagajewicz et al. (2000), focused on minimizing the operational and investment cost of a water network problem by using a tree search algorithm \[13\]; Liu et al. (2011) proposed a MILP problem by minimizing capital and operating costs applied to a city \[6\]. Finally, Rubio-Castro et al. (2012), proposed a
MINLP problem minimizing plant capital and piping operation costs, applied to an eco-industrial park\textsuperscript{[12]}.

On the other hand, other authors focus on the importance of the environmental impact generated by the water network. Boix et al. (2011) presents a MINLP problem applied to an IWN, where it seeks to minimize the flow of water extraction from natural sources \textsuperscript{[14]}. Mughees et al. (2013) seeks to increase the water efficiency of a petrochemical plant by minimizing its water consumption through a MINLP problem formulation \textsuperscript{[15]}. Finally, Hansen et al. (2018) also seeks to minimize the water consumption of a IWN by formulating an NLP problem \textsuperscript{[16]}.

Simultaneous minimization of environmental and economic impacts is less common. Kantor et al. 2015 developed a model to reduce network life cycle emissions while seeking to reduce their cost in an EIP network \textsuperscript{[17]}. Rojas-Torres et al. (2015) proposed a model to solve planning and scheduling water for a city, maximizing overall profit and minimizing fresh water consumption and land use \textsuperscript{[7]}. Pérez et al. (2019) proposed to design an optimal water distribution network maximizing revenues and minimizing both groundwater usage and investment cost \textsuperscript{[18]}.

The present research seeks to reduce the costs of installing treatment plants and operating the network, and to reduce the flowrate of extraction from natural sources, incorporating the economic and environmental impact.

None of the previously mentioned investigations present a multi-objective optimization problem on a city scale, considering the reuse of 4 different water qualities to supply different consumers. This way of approaching the problem, not considered before, is presented in this study. A novel superstructure is created for modeling a city-scale water system in order to plan new treatment plants and connections among stakeholders, taking into account simultaneous economic and environmental objectives.

The problem is formulated as a Multi-Objective Mixed-Integer Programming to decide the optimal configuration of a regional water system including environmental and economic considerations. The model is formulated to decide (i) the installation of new treatment plants, (ii) the actualization of the existing ones, and (iii) the connections within the new integration network. These changes on the water network allow to recycle and reuse water. The objective functions to minimize are the water usage from the source and the total cost of the water system. The problem is solved using the goal programming technique. The main novelties of this work are the large scale orientation of the formulation and the integration of economic and environmental objectives in the planning of a city-scale water system.

2. Problem Structure

The methodology used for this research is shown in Figure 1. Every step was followed by an analysis of the information or results.
In this problem, a city water system is modelled. This section describes the superstructure with all possible connections. The whole surface of the city is divided into sub-regions, based on population distribution. Participants are classified into (i) consumers; (ii) sources; (iii) distribution and collection nodes; (iv) treatment plants; and (v) final disposal sinks. Figure 2 shows the simplified graph.
Figure 2: Graph of the problem. Current and new plants are included as treatment nodes, and the respective consumption within a city.

There are two subsets of water sources and disposal sinks: surface and underground water. Consumption nodes represent a variety of uses: (i) domestic, (ii) commercial, (iii) industrial, (iv) agricultural, and (v) irrigation of urban areas. Each consumption node has its own demand. Some consumers require less restrictive qualities than drinking water.

Industrial consumption is subdivided in two subsets, depending on consumption magnitude. The first subset is large industrial consumers, which are particular big industries in the city. The second subset group, other companies represented as a cluster within each sub-region.

Concerning treatment nodes, different sets are created for existing and new treatment plants. Existing plants are in their original location. Also, all existing wastewater treatment plants have discharge quality water. Therefore, there is no recycling or reuse of water in the present system.

Four new sets of new wastewater treatment plants with different technological configurations were created. These can achieve four water qualities: (i) freshwater; (ii) drinking water; (iii) irrigation water; and (iv) discharge water. In addition, three new sets of modified treatment plants are created. Modified plants are existing plants modified to achieve a different output quality. These plants can achieve irrigation, water source, and drinking water quality.
DWTP and WWTP are subdivided into two subsets depending on their treatment capacity: large and small plants. Distribution and collection nodes aim at representing water distribution and collection networks of the current system. There is also a sink node to collect lost water from the nodes within the network.

The proposal model is a Mixed Integer Programming problem (MIP). The BARON solver was used, and the model was coded in the software GAMS.

3. Mathematical Model

The proposed model is based on the superstructure shown in Figure 2. The sets, variables and subscripts used in the model are defined in Nomenclature section.

Mass Balances

Water quality at the exit of each treatment plant and network consumption satisfy the quality constraints for their respective user. Stationary state is also assumed for each node. Finally, each flow density is assumed constant, then volumetric balances can be made.

Global Mass Balances

For the global mass balance it is assumed that all flows from surface and underground water sources are equal to the flows discharged into the environment plus the losses of each node that go into the sink node as shown in Equation 1.

$$\sum_{w} \sum_{i} F_{w \rightarrow i} = \sum_{j} \sum_{k} F_{j \rightarrow k} + \sum_{s} \sum_{s_{w}} F_{s \rightarrow s_{w}}, \forall w \in W, \newline \forall k \in K, \forall s \in S, \forall i \in \{CPW\}, \forall h \in \{WT\}, \forall j \in \{DNC\}$$  \hspace{1cm} (1)

In this equation, \(i\) corresponds to the water consuming nodes, \(j\) to those that discharge water into natural courses and \(h\) to all nodes which have water losses.

Mass Balances for each Node

At any node, the incoming flow rates will be the same as the outgoing flow rates, as a result of the steady state assumption. A general expression is shown in Equation 2.

$$\sum_{i \in ON} F_{i \rightarrow j} = \sum_{j \in DN} F_{j \rightarrow h} + \sum_{s_{w}} F_{j \rightarrow s_{w}}, \forall j \in \{AS_{j}\}$$  \hspace{1cm} (2)

In this equation, \(j\) corresponds to each node in which mass balance is carried out in all sets except water source, discharge, or sink, i.e. \(AS_{j}\); \(i\) represents the nodes of origin of the input flow rates, i.e., \(ON_{j}\); and \(h\) represents the destination nodes of the output flow rates, i.e., \(DN_{j}\).

Certain characteristics for consumption, treatment, distribution, and collection nodes are specified below.

a. Mass Balance for Consumption Nodes

Mass balance of residential and commercial consumption can be supplied with drinking quality. As all consumption and treatment nodes, there is a flow to the sink. This is expressed in Equation 3 and its representation is given by Figure 3.
Figure 3: Representation of the mass balance of commercial and residential consumption.

Local industrial consumption can be supplied only with drinking water quality. Therefore, mass balance is determined by the Equation 4 and its representation is given by Figure 4.

\[
\sum_{a_2 \in DS} F_{a_2 \rightarrow j} + \sum_{e_2 \in NS} F_{e_2 \rightarrow j} + \sum_{l_2 \in MA^2} F_{l_2 \rightarrow j} + \sum_{p_2 \in NA^2} F_{p_2 \rightarrow j} + \\
\sum_{d \in D} F_{d \rightarrow j} = \sum_{c \in C} F_{j \rightarrow c} + \sum_{b_2 \in WS} F_{j \rightarrow b_2} + \sum_{p_2 \in NA^2} F_{j \rightarrow p_2} + \\
\sum_{q_2 \in NB^2} F_{j \rightarrow q_2} + \sum_{r_2 \in NC^2} F_{j \rightarrow r_2} + \sum_{t_2 \in ND^2} F_{j \rightarrow t_2} + \sum_{l_2 \in MA^2} F_{j \rightarrow l_2} + \\
\sum_{m_2 \in MB^2} F_{j \rightarrow m_2} + \sum_{n_2 \in MC^2} F_{j \rightarrow n_2} + \sum_{s \in S} F_{j \rightarrow s} \quad \forall j \in \{RC \cup CC\}
\]

(3)

\[
\sum_{a_2 \in DS} F_{a_2 \rightarrow j} + \sum_{e_2 \in NS} F_{e_2 \rightarrow j} + \sum_{l_2 \in MA^2} F_{l_2 \rightarrow j} + \sum_{p_2 \in NA^2} F_{p_2 \rightarrow j} + \\
\sum_{d \in D} F_{d \rightarrow j} = \sum_{g_5 \in EB} F_{j \rightarrow g_5} + \sum_{s \in S} F_{j \rightarrow s} \quad \forall j \in IB
\]

(4)
Mass balances for large industrial consumers supplied with drinking water and discharging into surface water courses is presented in Equation 5 and its representation is shown in Figure 5. Mass balance for large industrial consumers supplied by fresh water and discharging to sewage system is presented in Equation 6 and its representation can be seen in Figure 6.

\[
\begin{align*}
\sum_{d \in D} F_{d \rightarrow j} + \sum_{a_2 \in DS} F_{a_2 \rightarrow j} + \sum_{e_2 \in ES} F_{e_2 \rightarrow j} + \sum_{p_2 \in NAA} F_{p_2 \rightarrow j} + \\
\sum_{l_2 \in MA} F_{l_2 \rightarrow j} &= \sum_{g_2 \in EL^dn} F_{j \rightarrow g_2} + \sum_{s \in S} F_{j \rightarrow s} , \forall j \in IL^dn
\end{align*}
\] (5)

\[
\begin{align*}
\sum_{w \in W} F_{w \rightarrow j} + \sum_{q_1 \in NB} F_{q_1 \rightarrow j} + \sum_{q_2 \in NS} F_{q_2 \rightarrow j} + \sum_{m_1 \in MB} F_{m_1 \rightarrow j} + \\
\sum_{m_2 \in MB} F_{m_2 \rightarrow j} &= \sum_{g_3 \in EL^fs} F_{j \rightarrow g_3} + \sum_{s \in S} F_{j \rightarrow s} , \forall j \in IL^fs
\end{align*}
\] (6)
Figure 6: Representation of the mass balance of large industrial consumption supplied with fresh water and discharging into sewage system.

Mass balance of the agricultural irrigation consumption nodes is given by the Equation 7 and its representation is shown in Figure 7.

\[
\sum_{w \in W} F_{w \rightarrow j} + \sum_{r_1 \in NC} F_{r_1 \rightarrow j} + \sum_{r_2 \in NC} F_{r_2 \rightarrow j} + \sum_{n_1 \in MC} F_{n_1 \rightarrow j} + \sum_{n_2 \in MC} F_{n_2 \rightarrow j} = \sum_{k_0 \in KG} F_{j \rightarrow k_0} + \sum_{s \in S} F_{j \rightarrow s}, \quad j \in AC
\]  

(7)

Figure 7: Representation of the mass balance of agricultural irrigation consumption.

Urban areas irrigation is subdivided in two subsets, depending on its source. The first one is supplied by drinking or fresh water; and the second, by fresh water. Mass balance of the first subset is given by the Equation 8 and its representation is given in Figure 8.
\[
\sum_{w \in W} F_{w \rightarrow j} + \sum_{r_1 \in NC^l} F_{r_1 \rightarrow j} + \sum_{r_2 \in NC^s} F_{r_2 \rightarrow j} + \sum_{n_1 \in MC^l} F_{n_1 \rightarrow j} + \\
\sum_{n_2 \in MC^s} F_{n_2 \rightarrow j} = \sum_{k_0 \in KG} F_{j \rightarrow k_0} + \sum_{s \in S} F_{j \rightarrow s}, \; \forall \; j \in PF
\]

(8)

Figure 8: Representation of the mass balance of urban park irrigation with freshwater consumption.

Mass balance of the second subset is given by the Equation 9 and its representation is shown in Figure 9.

\[
\sum_{a_2 \in DS} F_{a_2 \rightarrow j} + \sum_{d \in D} F_{d \rightarrow j} + \sum_{r_1 \in NC^l} F_{r_1 \rightarrow j} + \sum_{r_2 \in NC^s} F_{r_2 \rightarrow j} + \\
\sum_{n_1 \in MC^l} F_{n_1 \rightarrow j} + \sum_{n_2 \in MC^s} F_{n_2 \rightarrow j} = \sum_{k_0 \in KG} F_{j \rightarrow k_0} + \sum_{s \in S} F_{j \rightarrow s}, \; \forall \; j \in PD
\]

(9)

Figure 9: Representation of the mass balance of urban park irrigation with drinking or freshwater consumption.
b. Mass Balances of Treatment Nodes

For existing treatment plants, mass balance of large existing DWTPs is determined by Equation 10 and its representation is given by Figure 10.

\[
\sum_{w \in W} F_{w \rightarrow j} + \sum_{q_1 \in NB^I} F_{q_1 \rightarrow j} + \sum_{q_2 \in NB^S} F_{q_2 \rightarrow j} + \sum_{m_1 \in MB^I} F_{m_1 \rightarrow j} + \\
\sum_{m_2 \in MB^S} F_{m_2 \rightarrow j} = \sum_{d \in D} F_{j \rightarrow d} + \sum_{s \in S} F_{j \rightarrow s}, \ \forall \ j \in DL
\] (10)

Figure 10: Representation of the mass balance of a large existing drinking water treatment plant.

Mass balance for small existing DWTPs is given by Equation 11 and its representation is shown in Figure 11.

\[
\sum_{w \in W} F_{w \rightarrow j} + \sum_{q_1 \in NB^I} F_{q_1 \rightarrow j} + \sum_{q_2 \in NB^S} F_{q_2 \rightarrow j} + \sum_{m_1 \in MB^I} F_{m_1 \rightarrow j} + \\
\sum_{m_2 \in MB^S} F_{m_2 \rightarrow j} = \sum_{i_1 \in RC} F_{j \rightarrow i_1} + \sum_{i_2 \in CC} F_{j \rightarrow i_2} + \sum_{u_1 \in PD} F_{j \rightarrow u_1} + \\
\sum_{f_1 \in IL^{ds}} F_{j \rightarrow f_1} + \sum_{f_2 \in IL^{dn}} F_{j \rightarrow f_2} + \sum_{f_3 \in IB} F_{j \rightarrow f_3} + \sum_{s \in S} F_{j \rightarrow s}, \ \forall \ j \in DS
\] (11)

Figure 11: Representation of the mass balance for small existing drinking water treatment plant.
Mass balance of a large existing WWTP is given by Equation 12 and its representation is shown in Figure 12.

\[
\sum_{c\in C} F_{c\to j} = \sum_{k_1\in K} F_{j\to k_1} + \sum_{s\in S} F_{j\to s}, \; \forall \; j \in WL
\]  

(12)

Figure 12: Representation of the mass balance of a large existing wastewater treatment plant.

Mass balance of small treatment plants is similar, but the incoming flows come directly from consumers: commercial, residential, and industrial consumers that discharge into the sewer system.

New DWTPs allow to install a new plant in a different location. Mass balances for new large and small DWTPs are the same as existing ones.

New WWTPs can achieve four new output qualities: (i) drinking water, (ii) fresh water, (iii) irrigation water, and (iv) discharge in natural course. Their respective mass balances are determined by Equations 13, 14, 15 y 16, respectively and their respective representations are given by Figure 13, 14, 15 and 16 for each case.

\[
\sum_{c\in C} F_{c\to j} = \sum_{d\in D} F_{j\to d} + \sum_{s\in S} F_{j\to s}, \; \forall \; j \in NA^l
\]  

(13)

Figure 13: Representation of the mass balance of a large new wastewater treatment plant with drinking water output quality.
\[ \sum_{c \in C} F_{c \rightarrow j} = \sum_{a_1 \in DL} F_{j \rightarrow a_1} + \sum_{a_2 \in DS} F_{j \rightarrow a_2} + \sum_{f_3 \in ILF_s} F_{j \rightarrow f_3} + \sum_{f_4 \in ILF_n} F_{j \rightarrow f_4} + \sum_{s \in S} F_{j \rightarrow s}, \quad \forall \ j \in NB^l \] (14)

Figure 14: Representation of the mass balance of a large new wastewater treatment plant with fresh water output quality.

\[ \sum_{c \in C} F_{c \rightarrow j} = \sum_{h_1 \in AC} F_{j \rightarrow h_1} + \sum_{k_1 \in PD} F_{j \rightarrow k_1} + \sum_{u_2 \in PF} F_{j \rightarrow u_2} + \sum_{s \in S} F_{j \rightarrow s}, \quad \forall \ j \in NC^l \] (15)

Figure 15: Representation of the mass balance of a large new wastewater treatment plant with irrigation water output quality.

\[ \sum_{c \in C} F_{c \rightarrow j} = \sum_{k_1 \in K^s} F_{j \rightarrow k_1} + \sum_{s \in S} F_{j \rightarrow s}, \quad \forall \ j \in ND^l \] (16)
Modified wastewater treatment plants can be improved to achieve a different output quality: (i) drinking water, (ii) fresh water, and (iii) irrigation water. Their mass balances are the same as those of new WWTPs. For industrial wastewater treatment plants, the same logic as for their respective sources of consumption.

**c. Mass Balances for Distribution and Collection Nodes**

Distribution and collection nodes aim at representing the network of the water system. The mass balance of the distribution nodes is given by Equation 17 and its representation is shown in Figure 17.

\[
\sum_{a_1 \in DL} F_{a_1 \rightarrow j} + \sum_{e_1 \in NL} F_{e_1 \rightarrow j} + \sum_{p_1 \in NAI} F_{p_1 \rightarrow j} + \sum_{l_1 \in MAI} F_{l_1 \rightarrow j} = \\
\sum_{i_1 \in RC} F_{j \rightarrow i_1} + \sum_{i_2 \in CC} F_{j \rightarrow i_2} + \sum_{u_1 \in PD} F_{j \rightarrow u_1} + \sum_{f_1 \in IL_{ds}} F_{j \rightarrow f_1} + \\
\sum_{f_2 \in IL_{dn}} F_{j \rightarrow f_2} + \sum_{f_5 \in IB} F_{j \rightarrow f_5} + \sum_{s \in S} F_{j \rightarrow s}, \forall j \in D
\]  

(17)

\[\sum_{a_1 \in DL} F_{a_1 \rightarrow j} + \sum_{e_1 \in NL} F_{e_1 \rightarrow j} + \sum_{p_1 \in NAI} F_{p_1 \rightarrow j} + \sum_{l_1 \in MAI} F_{l_1 \rightarrow j} = \\
\sum_{i_1 \in RC} F_{j \rightarrow i_1} + \sum_{i_2 \in CC} F_{j \rightarrow i_2} + \sum_{u_1 \in PD} F_{j \rightarrow u_1} + \sum_{f_1 \in IL_{ds}} F_{j \rightarrow f_1} + \\
\sum_{f_2 \in IL_{dn}} F_{j \rightarrow f_2} + \sum_{f_5 \in IB} F_{j \rightarrow f_5} + \sum_{s \in S} F_{j \rightarrow s}, \forall j \in D
\]
\[ \sum_{i_1 \in RC} F_{i_1 \rightarrow j} + \sum_{l_2 \in ECC} F_{l_2 \rightarrow j} + \sum_{g_3 \in EL^d} F_{g_3 \rightarrow j} + \sum_{g_5 \in EL^e} F_{g_5 \rightarrow j} + \sum_{g_6 \in EB} F_{g_6 \rightarrow j} = \]
\[ \sum_{b_1 \in WL} F_{j \rightarrow b_1} + \sum_{p_1 \in NA^l} F_{j \rightarrow p_1} + \sum_{q_1 \in NB^l} F_{j \rightarrow q_1} + \sum_{r_1 \in NC^l} F_{j \rightarrow r_1} + \sum_{t_1 \in ND^l} F_{j \rightarrow t_1} + \]
\[ \sum_{l_1 \in MA^l} F_{j \rightarrow l_1} + \sum_{m_1 \in MB^l} F_{j \rightarrow m_1} + \sum_{n_1 \in MC^l} F_{j \rightarrow n_1} + \sum_{s \in S} F_{j \rightarrow s}, \forall j \in C \]

(18)

Figure 18: Representation of the mass balance of collection node.

**Covering the Demand**

To satisfy the water demand of each node, Equation 19 must be respected.

\[ \sum_{i \in ON_j} F_{i \rightarrow j} \geq DM_{\{j,p\}}, \forall j \in AD_j, \forall p \in CT \]

(19)

In this equation, \( i \) corresponds to nodes of origin of \( j \), i.e. \( ON_j \), \( j \) being the current node, i.e. where the demand must be satisfied \((AD_j)\), and \( p \) represent each district.

**Treatment Plant Capacity**

In all capacity restrictions there is a territorial association of plants and consumers sharing the plant district, i.e. the capacity of a plant in the \( p' \) district, shall meet the demands of \( p' \) district users.

**New Drinking Water Treatment Plant Capacity**

As mentioned above, there are large and small treatment plants. The value separating both categories varies with the consumption of each district. In the case of big new drinking water treatment plants Equation 20 must be respected.

\[ \sum_{d \in D} F_{i \rightarrow d} \geq j D M_{ij,p} \cdot E_i, \forall i \in NL, \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \]

(20)

On the other hand, for small new drinking water treatment plants, Equation 21 must be respected. A new parameter ‘\( m \)’, which corresponds to a small value in comparison to the demands of the district, is used to flexibilize the creation of small plants.
\[ \sum_{j} F_{i \to j} \leq \left( \sum_{j} D M_{(j,p')} + m \right) \cdot E_i, \quad \forall i \in NS, \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \]  \hfill (21)

**New Wastewater Treatment Plant Capacity**

For new big wastewater treatment plants, Equation 22 must be respected.

\[ \sum_{c} F_{c \to i} \geq \sum_{j} D M_{(j,p')} \cdot E_i, \quad \forall i \in \{NA^l \cup NB^l \cup NC^l \cup ND^l\}, \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \]  \hfill (22)

For small new wastewater treatment plant, Equation 23 must be respected.

\[ \sum_{j} F_{j \to i} \leq \left( \sum_{j} D M_{(j,p')} + m \right) \cdot E_i, \quad \forall i \in \{NA^s \cup NB^s \cup NC^s \cup ND^s\}, \forall p' \in CT \mid p' \text{ is the district where } i \text{ is positioned} \]  \hfill (23)

**Existing or Modified Treatment Plant Capacity**

Flow rates through existing or modified wastewater treatment plants must vary according to season. This fluctuation can be observed in Figure 19 and shows a higher consumption in summer than in winter. This logic is reflected in Equations 24 and 25. The percentages were obtained from the drinking water treatment plant in operation in Santiago.

![Drinking water consumption. Example](image)

*Figure 19: Drinking water consumption during a year. Illustrative example for the comprehension of the text. These values may vary each year.*

\[ \sum_{i \in ON} F_{i \to j} \geq 0.75 \cdot 0.85 \cdot AF_j \cdot E_j, \quad \forall j \in EW \]  \hfill (24)

\[ \sum_{i \in ON} F_{i \to j} \leq 1.23 \cdot AF_j \cdot E_j, \quad \forall j \in EW \]  \hfill (25)

$AF_j$ is the current flow rate treated by the plant, and $E_j$ corresponds to the existence of the plant $j$.

In the case of drinking water treatment plants, the analysis is similar as shown in Equations 26 and 27.

\[ \sum_{i \in DN} F_{j \to i} \geq 0.75 \cdot 0.85 \cdot AF_j \cdot E_j, \quad \forall j \in ED \]  \hfill (26)

\[ \sum_{i \in DN} F_{j \to i} \leq 1.23 \cdot AF_j \cdot E_j, \quad \forall j \in ED \]  \hfill (27)
Logical Relationships of Existence

If the plant does not exist, then the incoming flows must be zero. This can be written mathematically through Equation 28 applying the BigM method [19].

\[ \sum_i F_{i \rightarrow j} \leq M \cdot E_j \] (28)

In this equation, \( i \) belongs to the set of origin nodes of the treatment plant \( j \), i.e. \( ON_j, j \) corresponds to each wastewater treatment plant, \( M \) is a parameter, and \( E_j \) corresponds to the plant \( j \) existence.

In addition, there are also coexistence relations of plants: an existing and a modified plant cannot exist in the same place, as shown in the Equation 29.

\[ \sum_{j \in \{EW\cup MW\}} E_j \leq 1 \] (29)

Cost

The costs in the problem are divided in operational costs (OpC) and capital costs (CapC). OpC is given by the equation 30 and CapC is given by equation 31 [12]. The parameters used for the problem can be found in Appendix 1

\[ OpC = T \cdot E \cdot \sum_j \sum_i F_{i \rightarrow j} \cdot \left( g \cdot (h_j - h_i) + f \cdot \frac{\text{dist}(i,j) \cdot v^2}{2D} \right) \] (30)

\[ CapC = K_f \cdot E_i \cdot (\sum_i C_v r_i \cdot \sum_j F_{j \rightarrow i} + C_f r_i + T e r_i) \] (31)

Where 'OpC' is the operational cost of the system, 'T' is the annual operational time, 'E' is the cost per unit of electricity, 'f' is the Darcy factor, 'h' is the height of the node 'i', 'dist(i,j)' is the distance from node 'i' to node 'j', 'v' is the linear velocity, 'g' is the gravity acceleration, 'D' is the diameter of each pipe, 'K_f' is the factor to annualize cost, \( C_v r_i \) correspond to the investment cost parameter of plant 'i' which is multiplied by the summation of the flows entering plant 'i', 'C_f r_i' correspond to the intercept point in the ordinate, depending on the plant 'i', 'T e r_i' correspond to the terrain cost of the plant 'i', and 'F_{i \rightarrow j}' is the flow rate from node 'i' to node 'j'.

Objective Functions

The problem has two opposing objective functions to be minimized: water flow used from the water source (G0) and the total cost (TC). These two objective functions FO1 and FO2 can be represented by Equations 32 and 33, respectively.

\[ FO1 = \min \sum_{j \in CFW} F_{w \rightarrow j} = \min \ G0, \forall w \in W \] (32)

\[ FO2 = \min TC = \min \ (OpC + CapC) \] (33)

4. Multi-Objective Optimization Strategy

A goal programming method is applied to solve the MOO problem. The multi-objective formulation is shown in equation 34 subject to the constraints 35 and 36:

\[ \min \gamma \] (34)
\[
\frac{G_0-G_{0id}}{G_{0nid}-G_{0id}} \cdot w_1 \leq \gamma \tag{35}
\]

\[
\frac{TC-TC_{id}}{TC_{nid}-TC_{id}} \cdot w_2 \leq \gamma \tag{36}
\]

The indices \( id \) and \( nid \) correspond to ideal and non-ideal solutions, respectively. Parameters \( w_1 \) and \( w_2 \) represent the relative weights of each objective function. \( \gamma \) represents the variable to be minimized.

5. Case Study

The model presented was applied in the city of Santiago, capital of Chile. Santiago is Chilean political, economic, and institutional center. The city has a population of six million people \[^{[20]}\]. Santiago is located on the hydrographic basin of the Maipo River, where the main surface water sources are the Maipo and Mapocho Rivers. Santiago has been signed as a hydric stress zone, with an average regional deficit of 11.4\% \[^{[21]}\].

The system is represented by one large and one small drinking water treatment plant and one large and one small wastewater treatment plant. These 4 plants can treat all the real city flow. In the current system there is no water recycling, all wastewater treatment plants discharge their effluents into natural water courses.

A simplified version of the problem is presented. For simplicity, the city was divided into four areas as follows: North-East (NE or SL), North-West (NW or GF), South-East (SE or RC) and South-West (SW or HF). Population was distributed geographically according to the districts demographic information \[^{[20]}\]. This division is shown in Figure 20. In addition, only one big industrial consumption, 2 wastewater treatment plants and 2 drinking water treatment plants were considered. Consumption is the same as in real problem, since the districts were clustered only in fewer points, but with the same water use. The flow of the entire network is representative.

For characteristics of the different districts, including consumption, locations, costs, and losses, see Appendix 1.

![Division of Santiago for the simplified version. Modified image from INE \[^{[22]}\].](image)

\[^{[20]}\text{INE.}\]

\[^{[21]}\text{The average regional deficit of 11.4\% indicates...}\]

\[^{[22]}\text{INE (Instituto Nacional de Estadísticas).}\]
6. Results

The MIP model of the case study has 484 constrains, 1,075 variables (including 50 binary variables), and was executed in an INTEL CORE i7-7700 HQ computer with 16 GB of memory. After a CPU time of 0.125 seconds the result was 16.06 [m³/s] of flow extracted from the water source and a total cost of 945,932,577[USD/year], when the weights of both functions are equal.

With the results of Table 2, the multiobjective problem is formulated obtaining the Pareto curve shown in Figure 22, where the values at the extremes of the curve were removed to make other intermediate values more clearly visible. The complete Pareto curve is shown in Figure 21.

<table>
<thead>
<tr>
<th>Variable in objective function</th>
<th>Ideal value</th>
<th>Nonideal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate from water source: G0 (m³/s)</td>
<td>14.67</td>
<td>25.59</td>
</tr>
<tr>
<td>Total Cost: TC (USD/year)</td>
<td>940,797,564</td>
<td>980,974,542</td>
</tr>
</tbody>
</table>

Table 2: Results for mono-objective problems

Pareto curve for multi-objective problem

Figure 21: Pareto curve obtained for the multi-objective problem.
For equal importance in both objective functions, i.e., $w_1 = w_2 = 0.5$ in equations 35 and 36, the superstructure shown in Figure 24 is obtained. The mass balance for the complete system is shown in Figure 26. This can be compared to the current base case represented by Figure 23, where the overall mass balance is shown in Figure 25. The optimized configuration reduces the water extraction from the natural source in 35.7%. In this solution, the annualized total cost of the water network grows a 0.5% when compared with the solution at the economic extreme of the Pareto curve, when the importance of the economic objective function is complete.
Figure 23: Superstructure for the current situation of the city.
Figure 24: Superstructure for the optimized solution with equal weights for both objective functions.

Source: 25.00 [m³/s]  Sink: 8.72 [m³/s]  Discharge: 16.28 [m³/s]

Figure 25: Global mass balance for the current situation of the city.
If no new or modified plants are imposed on the model, then the solution is the point where $w_1 = 0$ and $w_2 = 1$ on the Pareto curve, i.e. the last point on the curve with the highest value of $G_0$ and the lowest value of $TC$. This partially represents the current situation, since, while the treated flow is the same as when the environmental indicator is not important, the cost is higher since flows are reconfigured within the network in a suboptimal way.

In summary, Table 3 shows a comparison between current system and optimized system.

<table>
<thead>
<tr>
<th>Plant type</th>
<th>Current system</th>
<th>Optimized system</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWTP</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>WWTP with discharge quality output</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>WWTP with potable quality output</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>WWTP with irrigation quality output</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

### 7. Discussions

With respect to the obtained results, the current network is not optimal for water treatment, under the assumption that both objectives have the same importance. The optimal network considers the installation of a new drinking water treatment plant and a new wastewater treatment plant. It is also observed the presence of large and small plants in the case of drinking and wastewater, using the established distribution and collection networks, respectively. Both existing wastewater treatment plants are modified, and a new small plant is installed. This is mainly due to the high costs of installing new plants, compared with the modification of existing ones. The modification of plants is realistic, since installation costs are high when compared with operation costs, because of land costs. This behavior approaches other international cases such as Singapore, where there are treatment plants on the roof of other existing plants [24].

The result includes small treatment plants. According to the model, these plants distribute or collect directly from their consumers. This is a simplification, as they do not connect to a particular type of consumer. However, these plants connect to a small district network, which in turn connects to individual consumers. For example, commercial and residential...
consumption, are found in all districts ($GF, HL, SL, RC$), however it is shown in Figure 24 that the modified WWTP with drinking quality output ($MWWTTP1_2$) feeds the $GF$ district consumption, while the distribution network feeds the remaining ones. Thus, the modified WWTP sends two flow rates, one towards commercial $GF$ consumption and the other towards residential $GF$ consumption: in order to implement, these flows are a single flow rate that is divided in the different consumption points, since they are in the same district.

The optimal result includes the installation of a new small DWTP instead of a new large DWTP, in order to supply drinking water consumers.

The water extracted is mainly used for irrigation and large industrial consumption, since there are no costs associated with transport, nor losses from water treatment. As there are other consumers to supply, WWTP needs to be installed or modified. There are two options: fresh water or drinking water output quality. With the first option, the flow must be connected to a DWTP plant for water treatment, so the costs and losses are higher. On the other hand, the second option allows large or small plants. The installation and modification of small WWTP reduces water losses associated with distribution and collection nodes.

Existing wastewater treatment plant was modified to supply irrigation water to different consumers. This quality allows to reduce water losses, compared to freshwater quality (which is connected to a DWTP, generating a extra loss for its treatment), or to the discharge quality (which is sent directly to natural courses).

Another interesting result is that no decision is made to install wastewater treatment plants with discharge output quality. This is because it would incur in high installation costs without being able to recycle the water. In addition, no decision is made to install wastewater treatment plants with natural source output quality either. This is interesting because in many countries, water is injected into the water table to refill it with naturally filtered water. Besides our model shows this solution as non-optimal in environmental and economic aspects, the advantage of this type of plant is social preference, as they make an indirect reuse of the water in the network.

The Pareto curve shows the solutions for objective functions when they have different combinations of relative importance. A trade-off of both functions is observed, where the relation between both functions is not direct. Moving through the Pareto curve, plants are installed or uninstalled, and this behavior is not linear. This causes some important differences in the costs of a point compared to its previous or subsequent point, visually this is observed as changes in the gradients of each point.

As a projection, it may be important to include the choice of different types of technologies to be installed as an optimization variable, not as a given parameter according to quality. Another combination of technologies could be economically or environmentally better depending on the regional context. It is also important to improve the districts configuration. This improvement can include a mapping of pipes in order to refine transport cost estimation within the districts.

Finally, it is planned to incorporate the limits of extraction flow through an analysis of the effects of Climate Change on different variables in Santiago. As mentioned before, the consumption of water can vary through years because of demographic pressure and Climate Change, and the sources of water could also change because of the same reason.
Conclusions

This paper deals with the management of water resources by integrating new water treatment plants to find the optimal configuration of the water network, applied to the case study of Santiago, Chile.

The characterization of the regional water use is addressed dividing the participants within the network in sets and defining parameters as demand, consumption, losses, location, and costs. This organized structure is an important framework to look for an improved configuration of the regional water network with a formal optimization technique.

An MIP was formulated with this purpose, where the discontinuous variables were given by the existence of drinking water treatment plants and wastewater treatment plants with different output qualities, to allow water recycling within the network. This decision of allowing different qualities for water within a system is important to assess and compare the significant number of possible recycling and reuse connections within a city.

The problem seeks to minimize the costs of installing plants and of transporting water as well as to minimize the flow extracted from the natural source.

To apply the methodology to Santiago, it was necessary to separate the city into four districts in order to establish the characteristics of the different nodes required. Pareto curve was found with optimal solutions for different weights of each objective function. In the case in which both functions have the same importance, some ideas can be listed as an heuristic to compare possible water networks in other cities: (i) it was found that the optimal solution involves installing a new drinking water treatment plant and a new wastewater treatment plant with drinking output qualities. This solution also proposes (ii) the modification of the two existing wastewater treatment plants to reach drinking and irrigation quality outputs respectively. These results allow a reuse of water within the network, where the modification of the current WWTPs is preferred, because of the high cost of installation of new plants. For plants with drinking water quality, (iii) the use of small plants is preferred since the losses generated by distribution/collection are reduced. These results show that (iv) it is more environmentally and economically convenient to reuse water for irrigation and drinking consumption rather than recycling water to the natural source.

Santiago can reduce the water extraction from the natural source in 35.7%. In this optimal solution, the annualized total cost of the water network grows a 0.5% when compared with the economic extreme of the Pareto curve.

Finally, the model can be implemented in other contexts, allowing better planning of water resource in any city. The proposed model is a formal strategy to help decision-makers to improve water resource planning for any city with simultaneous environmental and economic objectives.

Acknowledgments

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References


**Nomenclature**

**Parameters**
- \( DM_k \): Water demand of node \( k \), see table 4
- \( AF_i \): Current flowrate of existing plant \( i \), see table 5
- \( M \): Big \( M \), 1000000
- \( m \): Little \( m \), 0.02
- \( K_f \): Factor to annualize the capital cost, \( 1/20 \) \((year^{-1})\)
- \( T \): Annual operating time [h-year], 8760
- \( E \): Electricity cost [USD-kWh], 0.158
- \( f \): Darcy coefficient [], 0.009
- \( g \): Gravity acceleration [m-s\(^2\)], 9.8
- \( v \): Average velocity [m-s], 1.5

**Variables**
- \( F_{i,j} \): Water flow from node \( i \) to node \( j \)
- \( E_t \): Existence of node \( i \)
- \( \gamma \): Variable
- \( w_i \): Weight

**Sets**
- \( W \): Set for water sources
- \( ED \): Existing Drinking Water Treatment Plants
- \( EW \): Existing Waste-Water Treatment Plants
- \( IE \): Industrial Effluent Treatment Plants
- \( ND0 \): New Drinking Water Treatment Plants
- \( MW \): Modified Waste-Water Treatment plants
- \( NW \): New Waste-Water Treatment Plant
- \( RC \): Residential Consumption
- \( CC \): Commercial Consumption
- \( PC \): Urban Park Irrigation
- \( AC \): Agricultural Irrigation
- \( IC \): Industrial Consumption
- \( D \): Set for Distribution nodes
- \( C \): Set for Collection nodes
- \( K \): Set for discharge natural courses
- \( S \): Sink

**Subsets**
- \( W^G \): Groundwater sources
- \( W^S \): Surface water sources
**Subsets**

- **DL** Large Existing DWTPs
- **DS** Small Existing DWTPs
- **WL** Large Existing WWTPs
- **WS** Small Existing WWTPs
- **PD** Urban Park Irrigation with drinking water consumption
- **PF** Urban Park Irrigation with fresh water consumption
- **IL** Large Industrial consumers
- **IB** By-district Industrial consumers
- **EL** Industrial Effluent Treatment Plant connected with large industrial consumers
- **EB** Industrial Effluent Treatment Plant connected with by-district industrial consumers
- **NL** New Large DWTPs
- **NS** New Small DWTPs
- **MA** Modified WWTPs with drinking water output quality
- **MB** Modified WWTPs with fresh water output quality
- **MC** Modified WWTPs with irrigation water output quality
- **NA** New WWTPs with drinking water output quality
- **NB** New WWTPs with fresh water output quality
- **NC** New WWTPs with irrigation water output quality
- **ND** New WWTPs with discharge water output quality
- **KG** Natural groundwater discharge courses
- **KS** Natural surface discharge courses

**Subsubsets**

- **DG** Existing Small DWTPs with groundwater consumption
- **DS** Existing Small DWTPs with surface water consumption
- **ILd** Large Industrial consumers with drinking water consumption
- **ILf** Large Industrial consumers with fresh water consumption
- **ELd** Related to **ILd**
- **ELf** Related to **ILf**
- **MAl** Modified Large WWTPs with drinking water output quality
- **MBl** Modified Large WWTPs with fresh water output quality
- **MCl** Modified Large WWTPs with irrigation water output quality
- **MAp** Modified Small WWTPs with drinking water output quality
- **MBp** Modified Small WWTPs with fresh water output quality
- **MCp** Modified Small WWTPs with irrigation water output quality
- **NAl** New Large WWTPs with irrigation water output quality
- **NBp** New Large WWTPs with discharge water output quality
- **NCp** New Large WWTPs with discharge water output quality
The document discusses the formulation of sets and sub-subsets related to water systems, including water sources, distribution, collection, discharge, and consumption nodes. It introduces sub-subsets for different types of water usage scenarios, such as New Small WWTPs with drinking water output quality, New Small WWTPs with fresh water output quality, New Small WWTPs with irrigation water output quality, and New Small WWTPs with discharge water output quality.

Sub-subsets are defined for specific consumer types, such as large industrial consumers with drinking water consumption and sewage discharge, large industrial consumers with drinking water consumption and natural water course discharge, large industrial consumers with fresh water consumption and sewage discharge, and large industrial consumers with fresh water consumption and natural water course discharge. Each sub-subset is further related to other sets in the formulation, providing a structured approach to modeling different water management scenarios.

Other types of sets not included in the formulation include origin nodes for each mass balance in the node, destination nodes for each mass balance in the node, all set except for water sources, discharge natural courses or sink, sets consuming fresh water, sets that have water losses, sets that discharge water in natural courses, sets that include the consumption nodes, and sets representing each district.

General subscripts are also defined, including water source nodes, distribution nodes, collection nodes, discharge natural course nodes, residential consumption nodes, commercial consumption nodes, agricultural irrigation nodes, urban park irrigation nodes with drinking water consumption, urban park irrigation nodes with fresh water consumption, and large industrial consumption nodes with drinking water consumption and sewage discharge.

These definitions and formulations are essential for modeling and analyzing water systems, particularly in the context of wastewater treatment plants and their impact on drinking water, fresh water, irrigation water, and discharge water output qualities.
$f_2$ Large Industrial consumption node with drinking water consumption and natural water course discharge related to subsubsubset $IL^{dn}$

$f_3$ Large Industrial consumption node with fresh water consumption and sewage discharge related to subsubsubset $IL^{fs}$

$f_4$ Large Industrial consumption node with fresh water consumption and natural water course discharge $IL^{fn}$

$f_5$ By-district Industrial consumption node related to $IB$

$k_0$ Natural groundwater discharge courses related to subset $K^G$

$k_1$ Natural surface discharge courses related to subset $K^S$

**Treatment plants subscripts**

$a_1$ Large existing DWTP node related to subset $DL$

$a_2$ Small existing DWTP node related to subset $DS$

$b_1$ Large existing WWTP node related to subset $WL$

$b_2$ Small existing WWTP node related to subset $WS$

$g_1$ Industrial Effluent Treatment Plant node related to subsubsubset $EL^{ds}$

$g_2$ Industrial Effluent Treatment Plant node related to subsubsubset $EL^{dn}$

$g_3$ Industrial Effluent Treatment Plant node related to subsubsubset $EL^{fs}$

$g_4$ Industrial Effluent Treatment Plant node related to subsubsubset $EL^{fn}$

$g_5$ Industrial Effluent Treatment Plant node related to subset $EB$

$e_1$ New Large DWTP node related with subset $NL$

$e_2$ New Small DWTP node related with subset $NS$

$l_1$ Modified Large WWTP node with drinking water output quality related to subsubset $MA^l$

$l_2$ Modified Small WWTP node with drinking water output quality related to subsubset $MA^s$

$m_1$ Modified Large WWTP node with fresh water output quality related to subsubset $MB^l$

$m_2$ Modified Small WWTP node with fresh water output quality related to subsubset $MB^s$

$n_1$ Modified Large WWTP node with irrigation water output quality related to subsubset $MC^l$

$n_2$ Modified Large WWTP node with irrigation water output quality related to subsubset $MC^s$

$p_1$ New Large WWTP node with drinking water output quality related to subsubset $NA^l$

$p_2$ New Small WWTP node with drinking water output quality related to subsubset $NA^s$

$q_1$ New Large WWTP node with fresh water output quality related to subsubset $NB^l$

$q_2$ New Small WWTP node with fresh water output quality related to subsubset $NB^s$

$r_1$ New Large WWTP node with irrigation water output quality related to subsubset $NC^l$

$r_2$ New Small WWTP node with irrigation water output quality related to subsubset $NC^s$

$t_1$ New Large WWTP node with discharge water output quality related to subsubset $ND^l$
Appendix 1: Parameters for the case study

The consumption of the different districts are shown in Table 4, where the large industrial consumption is 1.18 m³/s, located in the southeastern district (SE). The treatment flows of the current treatment plants are shown in Table 5. In addition, factors related to water losses at each node are shown in Table 6. The locations of the elements are in Table 7. The average pipe diameters are shown in Table 8, the $C_{\text{vr}_i}$ parameter for the installation cost is shown in Table 9 and the $T_{\text{er}_i}$ parameter for the terrain cost, is given by Table 10.

Table 4: Consumption of the different districts ($D_{M_k}$)

<table>
<thead>
<tr>
<th>District</th>
<th>Residential demand ($m^3/s$)</th>
<th>Commercial demand ($m^3/s$)</th>
<th>Urban park irrigation ($m^3/s$)</th>
<th>Agricultural irrigation ($m^3/s$)</th>
<th>Industrial demand ($m^3/s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE</td>
<td>0.98</td>
<td>0.32</td>
<td>0.23</td>
<td>2.00</td>
<td>0.09</td>
</tr>
<tr>
<td>NW</td>
<td>3.52</td>
<td>0.31</td>
<td>0.27</td>
<td>0</td>
<td>0.38</td>
</tr>
<tr>
<td>SE</td>
<td>2.69</td>
<td>0.39</td>
<td>0.26</td>
<td>5.91</td>
<td>0.13</td>
</tr>
<tr>
<td>SW</td>
<td>2.76</td>
<td>0.42</td>
<td>0.31</td>
<td>0</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 5: Current flowrate of the treatment plants ($A_{F_i}$)

<table>
<thead>
<tr>
<th>Treatment plant</th>
<th>Present flowrate ($m^3/s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big drinking water treatment plant (located in SE)</td>
<td>10.53</td>
</tr>
<tr>
<td>Small drinking water treatment plant (located in NW)</td>
<td>5.38</td>
</tr>
<tr>
<td>Big wastewater treatment plant (located in NE)</td>
<td>7.76</td>
</tr>
<tr>
<td>Small wastewater treatment plant (located in SW)</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Table 6: Water loss factors in each set.

<table>
<thead>
<tr>
<th>Set</th>
<th>Water loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big drinking water treatment plant</td>
<td>0.05</td>
</tr>
<tr>
<td>Small drinking water treatment plant</td>
<td>0.2</td>
</tr>
<tr>
<td>Big wastewater treatment plant</td>
<td>0.05</td>
</tr>
<tr>
<td>Small wastewater treatment plant</td>
<td>0.2</td>
</tr>
<tr>
<td>Industrial wastewater treatment plant</td>
<td>0.05</td>
</tr>
<tr>
<td>Residential consumption</td>
<td>0.1</td>
</tr>
<tr>
<td>Commercial consumption</td>
<td>0.1</td>
</tr>
<tr>
<td>Urban park irrigation</td>
<td>0.55</td>
</tr>
<tr>
<td>Agricultural irrigation</td>
<td>0.66</td>
</tr>
<tr>
<td>Industrial consumption</td>
<td>0.15</td>
</tr>
<tr>
<td>Distribution</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Table 7: Locations of different districts, considering the lower left corner as the origin of the plan.

<table>
<thead>
<tr>
<th>Set</th>
<th>Element</th>
<th>Horizontal Position X [km]</th>
<th>Vertical Position Y [km]</th>
<th>Height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>Water source in SE</td>
<td>124</td>
<td>0</td>
<td>787</td>
</tr>
<tr>
<td>ED</td>
<td>Treatment plant in SE</td>
<td>113.6</td>
<td>10.3</td>
<td>649</td>
</tr>
<tr>
<td>ED</td>
<td>Treatment plant in SE</td>
<td>113.6</td>
<td>10.3</td>
<td>649</td>
</tr>
<tr>
<td>ED</td>
<td>Treatment plant in NW</td>
<td>20.6</td>
<td>103.3</td>
<td>501</td>
</tr>
<tr>
<td>RC</td>
<td>Consumption in NE</td>
<td>93</td>
<td>93</td>
<td>683</td>
</tr>
<tr>
<td>RC</td>
<td>Consumption in NW</td>
<td>31</td>
<td>93</td>
<td>683</td>
</tr>
<tr>
<td>RC</td>
<td>Consumption in SE</td>
<td>93</td>
<td>31</td>
<td>616</td>
</tr>
<tr>
<td>RC</td>
<td>Consumption in SW</td>
<td>31</td>
<td>31</td>
<td>507</td>
</tr>
<tr>
<td>CC</td>
<td>Consumption in NE</td>
<td>93</td>
<td>93</td>
<td>683</td>
</tr>
<tr>
<td>CC</td>
<td>Consumption in NW</td>
<td>31</td>
<td>93</td>
<td>683</td>
</tr>
<tr>
<td>CC</td>
<td>Consumption in SE</td>
<td>93</td>
<td>31</td>
<td>616</td>
</tr>
<tr>
<td>CC</td>
<td>Consumption in SW</td>
<td>31</td>
<td>31</td>
<td>507</td>
</tr>
<tr>
<td>PC</td>
<td>Consumption in NE</td>
<td>93</td>
<td>93</td>
<td>683</td>
</tr>
<tr>
<td>PC</td>
<td>Consumption in NW</td>
<td>31</td>
<td>93</td>
<td>683</td>
</tr>
<tr>
<td>PC</td>
<td>Consumption in SE</td>
<td>93</td>
<td>31</td>
<td>616</td>
</tr>
<tr>
<td>PC</td>
<td>Consumption in SW</td>
<td>31</td>
<td>31</td>
<td>507</td>
</tr>
<tr>
<td>AC</td>
<td>Consumption in NE</td>
<td>93</td>
<td>93</td>
<td>683</td>
</tr>
<tr>
<td>AC</td>
<td>Consumption in SE</td>
<td>93</td>
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Table 8: Average diameters for transport cost

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Table 9: $Cv r_i$ and $Cf r_i$ for each treatment plant

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<th>$Cf r_i [MUSD]$</th>
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Table 10: $T_{er_i}$ for each treatment plant