Title: Evaluating partial satisfaction of multiple SDGs

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Abstract

The 2030 Sustainable Development Agenda again actualizes the need to think about how goals as diverse as extreme poverty eradication, universal secondary education, reduced maternal mortality and climate change mitigation fit together. The standard approach to measuring development is to combine normalized indicators in different dimensions into a single composite index that weights each of these dimensions equally. Such indices have been used to rank countries, monitor trends, set policy priorities and evaluate development policies. This paper presents some arguments against composite indicators and proposes a theory of partitioned evaluation based on the principle of superiority in value that does not rely on aggregation to identify a maximally best set of alternatives. The basic assumption of this approach is that for each relevant value dimension, there are critical levels below which no matter how well we do in other value dimensions, this does not outweigh the loss in the dimension in question. We demonstrate that this approach differs significantly from other approaches that reject trade-offs such as with a Pareto criterion.

Introduction

The 2030 Sustainable Development Agenda again actualizes the need to think about how goals as diverse as extreme poverty eradication, universal secondary education, reduced maternal mortality and climate change mitigation fit together. This poses several challenges. First, these goals concern diverse objectives, the attainment of which is often measured on different scales and typically assessed in terms of "shortfalls" (Herlitz & Horan 2017). For example, household income must be at least US$1.90 (2011 purchasing power parity (PPP)); maternal deaths should be at most 70 deaths per 100,000 live births, etc. How might one aggregate diverse objectives like these? Second, what is the purpose of such an aggregation? Development indices have been used for at least two different purposes. First, such indices may be used for *descriptive* purposes to rank countries, monitor trends and identify priority areas. Second, an index may be used for *policy* purposes to set policy priorities and evaluate development policies. It is conceivable that the best aggregation depends on which purpose one has.

Third, *some* of these goals are strongly intertwined (Nilson, Griggs & Visbeck 2016, ICSU 2016). For instance, ending hunger in Sub-Saharan Africa interacts positively with poverty eradication, health promotion and universal education. Yet, food production requires a stable climate and interacts with climate change mitigation in several ways. How might one account for these effects? In this paper, we develop a framework for thinking about, and evaluating the goodness of, outcomes when there are multiple objectives that depend on each other, and present a way of evaluating non-ideal outcomes for policy purposes by partitioning the evaluation process.

In order to assess sustainable development in a country, it is desirable to rank outcomes in terms of the extent to which a given outcome fulfills the SDG Agenda. Such evaluations enable so-called *comparativist* approaches to decision making, according to which we ought to pursue the course of action that leads to the comparatively best outcome (Chang, 2013). This is particularly useful for policy makers. The question then arises: how should we rank outcomes that only partially satisfy the SDG Agenda?

The standard approach to measuring development is to combine normalized indicators in different dimensions into a single composite index. Prominent examples of this approach include the Human Development Index, Multi-dimensional Poverty Index, the Sustainable Development Goal Index (Deaton 2011; Alkire, Ballon & Foster et al. 2015; Sachs et al. 2016). To aggregate the indicators into a single composite index requires a weighting of different dimensions that implicitly allows for trade-offs between these dimensions (Alkire, Ballon & Foster et al. 2015). The standard approach is to ascribe equal weight to each dimension and to then apply a linear aggregation function (cf. Sachs et al. 2016; Prakash et al. 2017). Similar to GDP, the single value that such an aggregation yields may be used to ordinally rank outcomes and countries and as an object for optimization by policy makers.

We reject this approach on three grounds: 1) Normalization techniques involve implicit biases, and commonly-used aggregation functions such as the equal-weighting scheme are arbitrary (Sen 1999, Broome 2002). 2) Composite indicators require universal regularity in the relationships between goals. However, interactions between SDGs are often contextual (Nilson, Griggs & Visbeck 2016).[1] 3) Trading-off different goals allow for the possibility that a tremendously good result in some goals outweighs a catastrophic result in others. This is undesirable, particularly if there are discontinuities such as “tipping points” in economic, social or ecological systems. For example, promoting economic growth in LDCs is irrelevant if we fail to protect the land they live on from “sinking”.

Rather than searching for a single composite indicator, we propose to address how well multiple objectives have been met in a multi-step process by partitioning the outcome space (Ross 2015, Sen 1997). To generate such a partition, we adapt the idea of Superiority in Value (cf. Arrhenius & Rabinowicz 2015) to sustainable development. Superiority in Value captures the idea that trade-offs across value dimensions sometimes are problematic, but as we shall demonstrate, this approach differs significantly from other approaches that reject trade-offs, e.g. the Principle of Pareto efficiency. The basic assumption of this approach is that for each relevant value dimension, there are critical levels below which no matter how well we do in other value dimensions, this does not outweigh the loss in the dimension in question.

Critical levels of this kind can be justified in two main ways. First, there are some outcomes we want to avoid at all cost because they cancel or counteract the pursuit of other objectives. For example, improving energy access by using coal is of little value if we accelerate climate change and acidify the oceans. Second, there are some outcomes we want to prioritize because they facilitate or enable progress on other objectives. For example, educating girls in LDCs would reduce maternal mortality and help eradicate poverty. By first examining outcomes to avoid and outcomes which facilitate other goals, we

[1] For instance, “bioenergy production is widely assumed to counteract food security through land competition. But in the Nordic region, bioenergy markets have reinforced the agricultural and forest production systems — offering new and more diversified market opportunities and increasing farmers’ and forest owners’ resilience” (Nilson, Griggs & Visbeck 2016).

can work our way up, excluding outcomes, prioritizing important objectives and ways to achieve those objectives.

This evaluation process is more practical than the alternatives, takes irregularities into account, and respects contextual interaction between goals. In particular, the approach can be used to rule out outcomes where a tremendously good result on some objectives outweighs in a catastrophic result on other objectives.

An objection to the overall approach is that it, in some cases, will fail to fully determine a best outcome, and rather point to a set of best permissible outcomes that might be quite different in terms of how they do in the different goals. There are different ways to deal with this issue. A composite index could be invoked at this second step for example. As an alternative to equal weighting, one could assign extra weight to goals the poorer one does in that goal. Alternatively, one could dispense entirely with aggregation and focus on the partial ordering generated by the incomplete partition.

Amartya Sen (1997) address the problem of incompleteness in health equity evaluations as follows:

> "[O]ften enough health equity will yield an incomplete partitioning or a partial ordering. This does not do away with the discipline of rational assessment, or even of maximization (which can cope with incompleteness through reticent articulation), but it militates against the expectation, which some entertain, that in every comparison of social states there must be a full ranking that places all the alternative states in a simple ordering (Sen 1997: 3-4)."

Sen here claims that rationality only requires us to pick outcomes that are not worse than any alternative – and our model will always identify such alternatives provided a set of critical values has been set. Thus, this problem could be dismissed as relying on the false assumption that rationality requires us to choose outcomes that are "at least as good" as every alternative. It suffices to look at outcomes that are not worse than any alternative. Those two conceptions are of course identical if we have a complete ordering. But if we do not have that, as is typically the case with our proposal, the two conceptions come apart.

II: Accruing capped objectives

Assume that a set of capped objectives is relevant for outcome evaluations. In order to meet the overall objective, each separate objective must be fully met. Prima facie, for each objective, the better the objective is met, the better the outcome is in terms of the overall objective and the closer one is to fulfilling the overall objective. Such a set of objectives might include: 'household income must be at least US$1.90 (2011 purchasing power parity (PPP))'; 'maternal deaths should be at most 70 deaths per 100,000 live births; 'GHG emissions should be at most X tonnes'; etc. The overall objective might be the Sustainable Development Agenda. The question is: how do we establish which outcome is better when only partial satisfaction of the set of objectives is achieved?

The extent to which a capped objective has been met can be represented as the distance between a current level (CL) and a desired target level (TL), i.e. a shortfall. For example, a target level might be emissions of at most X tons of CO2 per year or that no individual lives in extreme poverty. How good an outcome is in terms of a capped objective can be expressed by the difference TL-CL. Thus, how well a CO2 emissions abatement objective has been met can (initially at least) be conceptualized as the current level of CO2 emissions abatement (CL_{CO2}) minus the target level of CO2 abatement TL_{CO2}: CO2 abatement shortfall = TL_{CO2}-CL_{CO2}. Thus, more generally, TL_O-CL_O = the objective shortfall. If an objective shortfall > 0, the outcome is bad in terms of the objective. If an objective shortfall ≤ 0, the objective is satisfied.

The extent to which a set of capped objectives has been met can be represented as a vector of shortfalls which allows us visualize different outcomes in terms of a set of capped objectives. Suppose we face the problem of ranking the following alternative outcomes such that:

	Objective X	Objective Y	Objective Z
Outcome A:	5	7	3
Outcome B:	0	14	0
Outcome C:	1	3	7

Without committing to anything except that shortfalls should be as small as possible, i.e. assuming that smaller shortfalls are always better, we can see that outcome A is better than outcome B in terms of objectives Y, but worse in terms of objective X and Z. Outcome A is better than outcome C in terms of Z, but worse in terms of X and Y. And B is better than C in terms of X and Z, but worse in terms of Y. This is interesting, but it does not answer the question that we really need an answer to: which of two (or more) outcomes is the better?

A first question that arises here is: can outcome vectors at all be ordinally ranked in terms of what matters?[2] That *no* ordinal rankings are ever possible when outcomes are evaluated in terms of a set of capped objectives is an implausible view. To see this, imagine that there is a fourth outcome in the example above, B':

	Objective X	Objective Y	Objective Z
Outcome A:	5	7	3
Outcome B:	0	14	0
Outcome B':	0	13	0
Outcome C:	1	3	7

Surely, we have no problems seeing that B' is better than B. B' is equal to B in terms of Objectives X and Z, and better than B in terms of Objective Y. Surely, we can be certain that B' > B if we know that Objectives X, Y and Z are all that matters (cf. Herlitz, 2016; Vallentyne & Tungodden, MS).

A second question can here be posed: what basis should one use for ordinal rankings of different outcome vectors in terms of what matters? Two important issues arise in relation to this second question: First, what, if any, is an appropriate *unit of measurement* to express total shortfall in? Secondly, what, if any, is an appropriate way of *weighting* different shortfalls? In the following section, we consider two possible approaches to aggregation of multiple capped objectives. In the fourth section, we present a way of thinking about outcome evaluations that is essentially non-aggregative.

III: Aggregating capped objectives

First, it could be suggested that there is some unique, precise covering value that can be invoked in order to weigh partial satisfaction of different objectives. A covering value is a

[2] Social choice theory has taught us that there are situations in which this is sometimes impossible, e.g. when one attempts to aggregate social preferences while respecting a set of highly plausible criteria (Arrow, 1951). Recent research in moral philosophy gives us further reasons to suspect that we might encounter severe difficulties when we to ordinally rank outcomes in moral-philosophical areas (Arrhenius, 2000; Chang, 2002; Parfit, 1984; Temkin, 2012).

value in terms of which outcomes can be compared such that if $ was a valid covering value, the goodness of all outcomes could be expressed in $, and eventually ranked in terms of how much $ they generate. Numerous normative theories can be understood as providing us with such a covering value. Such theories include utilitarianism (total sum of utility), prioritarianism (total weighted utility), (monistic) sufficientarianism (e.g. total utility to group under threshold), and well-specified versions of the capabilities approach. Outcomes A, B and C can, in other words, for example be evaluated and ranked in terms of how much utility is produced, according to a prioritarian calculus, according to sufficientarian cover values, or in terms of a well-specified capabilities approach-covering value. In the literature on development indices, the standard approach here is to normalize each indicator or objective by rescaling it from a lower bound set equal to 0 to an upper bound set equal to 100, with 100 being the best the score and 0 the worst score. The best score may be the SDG target itself or the average of some subset of the top performing regions. The normalized value then becomes the unique covering value for aggregation.

Second, once a common unit of measurement have been determined for each value dimension, it could be proposed that there is some unique aggregation which is relevant to the evaluation and that we ought to aggregate multiple objectives by some weighting scheme that implicitly allows for trade-offs between dimensions (Alkire Ballon & Foster et al. 2015). For example: ‘maximize 3X + 2.4Y + Z’. Functions of this kind can of course be much more complex than the simple linear form suggested here. Yet, regardless of the degree of complexity, they all provide us with a way in which to ordinally rank outcome vectors via some continuous function that aggregates the relevant objectives *f*(X, Y, Z). No single aggregation is currently explicated in the literature on development indices. The standard approach is to ascribe equal weight to each goal (cf. Sachs et al. 2016; Prakash et al. 2017). If we evaluate outcomes A, B and C using a precise covering value, objectives X, Y and Z are not used as grounds for evaluating the goodness of outcome any longer. They are merely a detour, and it would make more sense to simply look at outcomes A, B and C from the perspective of the covering value, and directly compare them in terms of how well they fare in terms of the covering value. In other words, resorting to a precise covering value entails making regularity assumptions that are unwarranted.

Secondly, approaches that invoke a single covering value and linear aggregation of objectives have difficulties accounting for what Frances Kamm has called ‘contextual interaction’ (Kamm, 1993; Temkin 2012). The (negative) value of an outcome’s shortfall P might be dependent on outcome shortfalls in other objectives, and vary depending on context. In the area of sustainable development, this is very easy to see (cf. Nilson, Griggs & Visbeck 2016). Increased frequency of extreme weather from global climate change is worse if it affects a poor country than a rich country. Aggregation functions could, in principle, take these interdependencies into account. Yet, most approaches that attempt to develop composite indicators assume additive separability of the indicators, which renders it impossible to take these interdependencies into account (cf. Broome, 1991; Kagan, 1987).

Thirdly, trading-off different goals allows for the possibility that a tremendously good result in some dimension outweighs a catastrophic result in others. It is undesirable to rely on approaches that have built-in assumptions that make this possible, since it is not at all clear that we should allow for this, particularly if there are discontinuities and “tipping points” in economic, social or ecological systems. For example, promoting economic growth in LDCs seems irrelevant if we fail to protect the land they live on from ‘sinking’. Consider the following two outcomes,

	Objective X	Objective Y	Objective Z
Outcome A:	5	7	3
Outcome B:	0	14	0

Note that we can see that outcome A is substantially better than outcome B in terms of objective Y, but considerably worse in terms of objective X and Z. Suppose Y = 10 is a tipping point such that if Y ≥ 10, then a catastrophic cost is imposed on the country. If we were to simply take an average of the values for these objectives for each outcome, which is roughly what most development indices currently do, it would appear that outcome B is better than outcome A.

We thus reject this general, aggregative approach to how to accrue multiple capped objectives on three grounds: they rely on regularity assumptions that are unwarranted; they fail to recognize the importance of contextual interaction; and they ignore the issue of tipping point. In the following section, we will present a non-aggregative approach to how to accrue multiple capped objectives and claim that this is a more plausible approach to these issues.

IV: Partitioned outcome vector evaluations

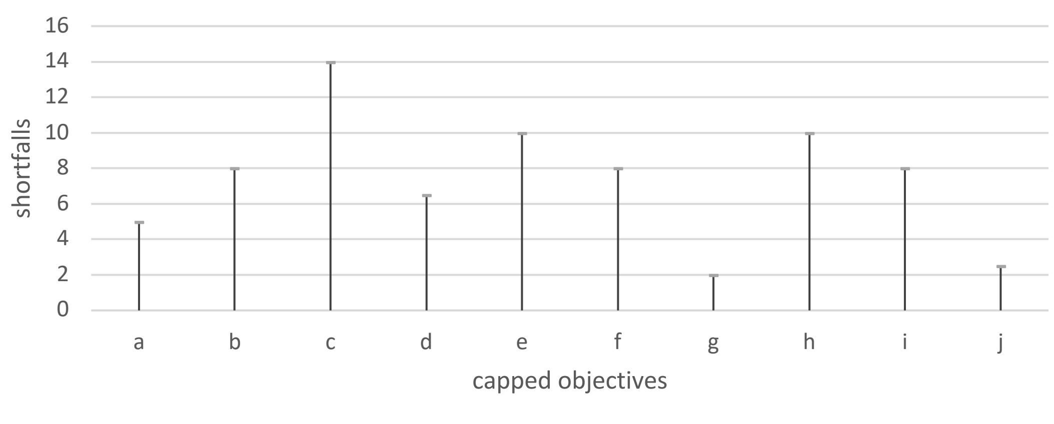
In this section, we introduce an alternative way of thinking about outcome evaluations for situations in which there are multiple capped objectives. A particular benefit of our approach is that it is neutral with respect to completeness. It can generate or approximate complete rankings, but it does not have to. We suggest that rather than striving for a complete ranking of outcome vectors, progress can be made by thinking about the conditions under which outcome vectors are superior to other outcome vectors.

4.1 The Model

Suppose $N = \{1,2, \dots, n\}$ is a set of capped objectives, in which each objective $i \in N$ is defined by a target level, denoted $t_i > 0$, and an achieved level, denoted $c_i \geq 0$. We can define the shortfall for objective i, denoted g_i, as the difference between the target level and achieved level which is given by $g_i = t_i - c_i$. We allow for the fact that different objectives may require different units of measurement and for simplicity at this point, we assume that for each objective, achieved levels do not exceed target levels, i.e. $c_i \leq t_i$ and thus $g_i \geq 0$.

We define the outcome space, denoted $\mathcal{G}$, as the set of all possible shortfalls which is given by $\mathcal{G} = \{g \in \mathbb{R}^n : g = (g_i)_{i=1}^n, g_i \geq 0\}$. The overall objective is defined as bringing the achieved level for each objective up to its target level, i.e. eradicating all shortfalls or "closing the gaps" for each objective. In practice, resource or other constraints may allow for only partial satisfaction of the overall objective.

Fig. 1: Mulitple capped objectives



4.2. Principle of Pareto Efficiency

In line with the discussion of previous sections, we shall assume that a smaller shortfall in any dimension, ceteris paribus, is better with respect to the overall objective. This assumption enables us to introduce the Principle of Pareto efficiency for multiple capped objectives. Suppose $g, g' \in \mathcal{G}$ are two outcomes, then g is Pareto superior to g' provided that $g_i \leq g'_i$, for all $i \in N$ and $0 \leq g_j < g'_j$ for some $j \in N$. The restriction $0 \leq g_j$ takes account of the capped nature of the objectives, and can be interpreted as a type of focus axiom, similar to those used in poverty measurement, namely that exceeding the target level of a specific objective adds not extra value to the overall objective of closing all gaps.

This principle of Pareto efficiency yields a partial ordering over the outcome space $\mathcal{G}$, without specifying any aggregation of the different shortfalls. In particular, having a lower shortfall in some objective is an improvement. It is clear this partial ordering is a minimal criterion, and it does not yield a complete ordering of the outcome space. Although there is a unique Pareto optimal outcome, given by $g = 0$. This outcome is unattainable when only partial satisfaction of the overall objective is possible. It does not allow us to define a unique best outcome for all possible subsets of the outcome space.

4.3. Principle of Superiority in Value

We now define an alternative partial ordering using the idea of side constraints and the Principle of superiority in value (cf. Arrhenius & Rabinowicz 2015). The basic assumption of this approach is that for each outcome vector and each relevant objective, there might be threshold levels below which no matter how well we do in the other objectives, this does not outweigh the shortfall for the objective in question.

More formally, suppose for some objective $i \in N$, $g_{i1} \geq 0$ is a threshold point in dimension *i* of degree 1 such that for any two outcomes $g, g' \in \mathcal{G}$, we have that g is superior in value to g' in objective *i* if and only if $g_i \leq g_{i1}$ and $g'_i > g_{i1}$. The superiority condition states that any outcome vector characterized by a shortfall in objective *i* that is lower than the threshold point in that dimension is superior to any outcome vector whose shortfall in objective *i* exceeds this threshold, irrespective of the relative sizes of these vectors' shortfalls in other dimensions.

If $g_{i1} > 0$, then we say that g is weakly superior in value to g' in objective *i* since *some* amount of shortfall in dimension *i*, below the threshold level, dominates all levels of shortfalls in other dimensions. If $g_i(1) = 0$, then we say that g is strongly superior in value to g' in objective *i* since *any* amount of shortfall in objective *i* dominates all levels of shortfalls in other dimensions.

Since threshold points can be defined in other dimensions as well, suppose $C_1^* = \{g_{i1}: i \in K_1 \subseteq N, |K_1| = k_1 \leq n\}$ is the set of threshold points of degree 1 for the number of dimensions k_1 in which these are defined. For any two outcomes $g, g' \in \mathcal{G}$, we say that g is superior in value to g' in degree 1 if and only if $g_i \leq g_{i1}$ for all $i \in K_1$ and $g'_j > g_{j1}$ for some $j \in K_1$. This superiority condition states that any outcome vector that satisfies all the thresholds is superior to any outcome that violates one or more of these thresholds, irrespective of size of the shortfalls in other dimensions.

Finally, for any two outcomes $g, g' \in \mathcal{G}$, we say that g is weakly superior in value to g' in degree 1 if and only if for *all* dimensions $i \in K_1$, g is weakly superior to g' in objective *i*. Similarly, g is strongly superior to g' in degree 1 if and only if for all dimensions $i \in K_1$, g is strongly superior to g' in objective *i*.

4.4. Partitioning the Outcome Space

Having defined the superiority relation at points in $\mathcal{G}$, we can now extend the concept to subsets of $\mathcal{G}$. Let $\mathcal{G}_1 = \{g \in \mathcal{G}: g_i > g_{i1}, g_{i1} \in C_1^*\}$ denote the set of all outcome vectors with the property that their shortfall in some objective *i* exceeds the relevant threshold in that objective, and let $\mathcal{G}/\mathcal{G}_1$ denote the complement of this set, i.e. the set of outcomes that satisfy all threshold points of degree 1. It is easily verified then that every element in $\mathcal{G}/\mathcal{G}_1$ is superior in value to every element in $\mathcal{G}_1$, and we can write $\mathcal{G}/\mathcal{G}_1 > \mathcal{G}_1$.

This process can be repeated on $\mathcal{G}/\mathcal{G}_1$, provided a well-defined set of threshold points in degree 2 is available. Given these threshold points, the iteration of this process will then identify two mutually disjoint subsets, $\mathcal{G}/\mathcal{G}_2$ and $\mathcal{G}_2$, that partition $\mathcal{G}/\mathcal{G}_1$ such that $\mathcal{G}/\mathcal{G}_2$ is superior in value to $\mathcal{G}_2$.[3]

The iteration of this process for threshold points of differing degrees thus generates a sequence of subsets $(\mathcal{G}_p)_{p=1}^{P}$ of $\mathcal{G}$ which are pairwise disjoint, i.e. $\mathcal{G}_p \cap \mathcal{G}_r = \emptyset$, for all $p \neq r$, and their union covers the entire outcome space, i.e. $\mathcal{G} = \cup_{p=1}^{P} \mathcal{G}_p$. The sequence thus partitions the outcome space, and number of degrees *P* determines the length of the sequence and the size of the partition on $\mathcal{G}$. Finer partitions of $\mathcal{G}$ can only be obtained by defining further sets of threshold points.

4.4.1. Totally Ordering the Partition

The corresponding equivalence relation, denoted $\sim$, can be defined as $g \sim g'$ if and only if $g \in \mathcal{G}_p$ and $g' \in \mathcal{G}_p$ and we refer to equivalence classes of this relation as permissible sets. Since every element of $\mathcal{G}_p$ is superior in value to every element $\mathcal{G}_{p-1}$, for all $p = 1,2,..,P$, the outcome space is thus partitioned according to the superiority in value relation. In particular, for all $g \in \mathcal{G}_p$ and $g' \in \mathcal{G}_r$, we have that $g > g'$ if $p > r$, and consequently the partial ordering of $\mathcal{G}$ by $>$ induces a total ordering of the set of permissible sets such that for any $\mathcal{G}_p, \mathcal{G}_r \in (\mathcal{G}_p)_{p=1}^{P}$, $\mathcal{G}_p > \mathcal{G}_r$ if and only if $p > r$ and we refer to $\mathcal{G}_p$ as the permissible set of degree *p*.

4.4.2. Maximization

The overall objective can then be approached using the superiority in value relation by seeking to identify the maximally attainable set that resources and other constraints allow. In light of these constraints, there will be a unique p^* such that $\mathcal{G}_{p^*}$ is the maximally attainable set, in the sense that $\mathcal{G}_{p^*} > \mathcal{G}_r$ for all $r < p^*$ where $(\mathcal{G}_p)_{p=1}^{p^*}$ are the attainable sets, and $\mathcal{G}_r > \mathcal{G}_{p^*}$ for $r > p^*$ are unattainable sets.

4.5. Pareto Efficiency and Superiority in Value

If one outcome is Pareto superior to another, then it is not necessarily true that it is superior in value. This depends on the threshold points and whether the Pareto superior outcome violates the relevant threshold. On the other hand, if an outcome is superior in value to another, a sufficient condition for Pareto superiority is that the threshold points in that degree are defined for *all* objectives. In situations where there are many capped objectives and threshold points for each degree are defined in some but not all dimensions, it does not necessarily follow that an outcome which is superior in value is a Pareto improvement over

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[3] For clarity of exposition and to provide precise definitions of relevant concepts, we describe here the *p*-th iteration of the process. Suppose the process has been iterated *p-1* times, where $p > 1$, and let $C_p^* = \{g_{ip}: i \in K_p \subseteq N, |K_p| = k_p \leq n\}$ denote a set of threshold points of the *p*-th degree. Each threshold point is non-negative, i.e. $g_i(p) \geq 0$, and lower than any threshold point defined in that objective in previous iterations, i.e. for each $i \in K_p$, $g_{ip} < g_{ir}$, if $i \in K_r$ for any $r = 1,2,..,p-1$. For any two outcomes $g, g' \in \mathcal{G}$, we have that g is superior in value to g' if and only if $g_i \leq g_i(p)$ for all $g_{ip} \in C_p^*$ and $g'_j > g_{jp}$ for some $g_{jp} \in C_p^*$. Let $\mathcal{G}_p$ denote the set of outcome vectors that violate one or more of the threshold points of degree *p*. Taking the complement of this set, we have that every element in $\mathcal{G}/\mathcal{G}_p$ is superior in value to every element in $\mathcal{G}_p$, and we can write $\mathcal{G}/\mathcal{G}_p > \mathcal{G}_p$.

it. Once a maximal set has been identified, the Pareto principle can be used to implement a partial ordering over that set.

4.6. Piecewise SDG Indices

One advantage of partitioned evaluation is that one can define piecewise SDG indices on the partitioned subsets. Piecewise indicators ordered with respect to superiority in value allow for outcome evaluations that take account of critical points and discontinuities in value. Given a partition $(\mathcal{G}_p)_{p=1}^{P}$ of $\mathcal{G}$, suppose $v: \mathcal{G} \to \mathbb{R}$ is a piecewise index such that sub-functions are defined for all partitioned subsets, i.e. $v_p: \mathcal{G}_p \to \mathbb{R}$ for all $p = 1,2,..P$. Thus if $p > r$, then all outcomes in $\mathcal{G}_p$ are superior in value to all outcomes in $\mathcal{G}_r$ and we say v_p is superior in value to v_r. Weighting schemes for sub-indicators could be set in way that better reflects the development imperatives of the country by taking account of their stage in the development process. Moving from permissible sets of lower degree to higher degree can permit a re-evaluation of normative considerations.

VI: Illustration

Let us reconsider the situation illustrated in Section III. The purpose of this illustration is to show how partitioned approach can be used to eliminate an outcome where a tremendously good result on some objectives outweighs a catastrophic result on one objective that would be ranked higher in a standard composite approach. Recall the following two outcomes:

	Objective X	Objective Y	Objective Z
Outcome A:	5	7	3
Outcome B:	0	14	0

Note that outcome A is substantially better than outcome B in terms of objective Y, but considerably worse in terms of objective X and Z. Suppose Y = 10 is a tipping point such that Y ≥ 10 imposes a catastrophic cost on the country.

If we were to simply take an average of the values for these objectives for each outcome, which is roughly what most development indices currently do, then outcome B is better than outcome A and the good results on objectives X and Z would outweigh the catastrophic result in outcome Y.

To avoid this type of scenario, we can use partitioned evaluation according to superiority in value. Suppose the set of all possible shortfalls is given by $\mathcal{G} = \{(x, y, z) \in \mathbb{R}^3 : x, y, z \geq 0\}$. Suppose Y = 10 is a threshold point of degree 1 in dimension y. Given this threshold, let $\mathcal{G}_1 = \{(x, y, z) \in \mathcal{G}: Y \geq 10\}$ denote the set of outcomes that violate this threshold and let $\mathcal{G}/\mathcal{G}_1$ denote the set of outcomes that respect this threshold. Then $\mathcal{G} = \mathcal{G}/\mathcal{G}_1 \cup \mathcal{G}_1$ and $\mathcal{G}/\mathcal{G}_1$ is superior in value to $\mathcal{G}/\mathcal{G}_1$. Since $A \in \mathcal{G}/\mathcal{G}_1$, $B \in \mathcal{G}_1$, and $g_{Y1} > 0$ we see that outcome A is weakly superior to outcome B. Thus, outcome B cannot be maximal and we eliminate outcome B from the evaluation provided A is an attainable outcome.

VII: Concluding Remarks

In this paper, we have addressed the problem of outcome evaluations when there are multiple capped objectives. We first showed some shortcomings with approaches that rely on the standard composite indicator. Thereafter, we presented a different model for how to think about such evaluations that avoids these shortcomings. This model invokes superiority in value and how this might occur when one compares outcome vectors. We suggested that an appropriate approach identifies a set of maximal outcomes. This will not always allow one to identify a best outcome, but it better respects the objectives that are accrued. In situations where no best outcome can be identified, an approach to the third step needs to be adopted. Such an approach could advance the maximization of a piecewise SDG index.

A particular issue that arises in relation to our suggestion, and that we have not said much about, is: Where do thresholds come from? This issue can be approach in at least two different ways. First, it could be argued that such thresholds exist in an objective sense. One could argue that the thresholds are instrumental, and that they can be inferred from the badness that follows if they are breached. It is, for example, claimed that if the glaciers melt beyond a certain level, it will have very bad consequences. Secondly, it could be argued that the thresholds are created through societal commitments. Such commitments can be the result of a deliberative process among multiple stakeholders. Decisions made through just processes in a democratic society ought to have some import on outcome evaluations. One way of seeing this sort of influence over outcome evaluations is in terms of thresholds posed on a set of complexly interrelated objectives.

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