Global Inequality and Transboundary Pollution

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Abstract

Global inequality is a major concern that surrounds the international political economy of climate change. In this paper, we analyze the response of the path of the transboundary pollution stock to a reduction in global inequality among countries in a framework that allows countries to make their environmental policy strategically. We show that reducing global inequality would have a mitigating effect on the path of the transboundary pollution stock if the relative number of richly-endowed countries in the subgroup of high-tech countries is greater that the relative number of richly-endowed countries in the subgroup of low-tech countries. Viewed from a different perspective, the response of the transboundary pollution stock path to a reduction in global inequality results from the combined action of two opposite forces. The mitigating force can be associated with a bi-dimensional measure of both richly-endowed/high-technology countries and poorly-endowed/low-technology countries. The aggravating force can be associated with a bi-dimensional measure of both richly-endowed/low-technology countries and poorly-endowed/high-technology countries.

Keywords: Global Inequality, Pollution Stock Path, Climate Change, International Political Economy, Technology, Poor and Rich Countries, International Redistributive Policy.

JEL Classification: D63, D62, Q5

1 Introduction

Global inequality has become a fundamental consideration at every major conference on climate change. The 2015 Paris Climate Summit reemphasized that developed countries should continue to offer financial support to help developing countries reduce emissions and adapt to climate change. Developed countries had previously pledged to provide 100 billion by 2020 in this regard. This reflects the idea that reducing income inequality among countries will help mitigate the path of the future pollution stock and lead to an improved environment quality. The paper by Holtz-Eakin and Selden (1995) emphasizes the importance of exploring the political economy of distribution as it relates to climate policies. As they put it, policy analysis is needed to avoid conflicts between mitigating climate change and improving the global distribution of income. The aim of this paper is to explore these ideas by providing a framework that helps understanding some crucial factors that may play out behind the response of the transboundary pollution stock path to a reduction in global inequality.

This paper approach differs from the previous literature by analyzing the effects of redistributing a non-polluting endowment from rich countries to poor countries on the path of the future pollution stock in a strategic dynamic game model of transboundary pollution. To be clear, this paper does not look at the general equilibrium effects of redistributing a pigouvian tax aiming at internalizing the negative externality, but rather we clarify the conditions under which redistributing an existing non-pollution endowment from rich countries to poor countries will mitigate the path of the transboundary pollution stock. Our work is related to, at least, two strands of the literature. One is devoted to the general equilibrium effects of redistribution and another one is devoted to the strategic analysis of transboundary pollution with multi-country dynamic models of stock pollution. There is a vast body of work in these areas and a comprehensive review is well beyond the scope of this paper. Instead, we provide a selective review which highlights our contribution. In analyzing pollution problems, the literature devoted to the general equilibrium effects of redistribution mostly focuses on emission (pigouvian) taxation schemes that sustain Pareto efficient outcomes (Lange and Requate, 2000; Benchekroun and Van Long, 1998; Goulder, 1995; Lans Bovenberg and de Mooij, 1994; Stavins, 2011; Martin et al., 1993; Williams et al., 2015). For instance, Lange and Requate (2000) analyze the existence of a first-best pigouvian tax when the redistribution rule is fixed in a production-consumption model while Benchekroun and Van Long (1998) analyze a dynamic game in which the transboundary pollution stock is contributed by oligopolists in order to find an emission tax rule that will achieve the socially optimal pollution stock path. Along the same lines, the broad literature focused on environmental and climate justice has advocated for a reduction in inequality as a tool to adress environmental problems (Shue, 1999; Parks and Roberts, 2008; Roberts, 2001; Boyce, 1994). The question of inequality and redistribution has also received a substantial attention from the macroeconomic literature that looks at economic growth, however without giving enough attention to transboundary pollution issues and climate change (Barro, 2000; Aghion et al., 1999; Piketty, 2014; Dabla-Norris et al., 2015; Forbes, 2000; Galor and Zeira, 1993). The literature devoted to transboundary pollution problems and multi-country dynamic models of stock pollution uses a game theoretical framework to express important strategic interactions between countries in addressing the problem of climate change (Benchekroun and Chaudhuri, 2014; Dockner and Long, 1993; Jørgensen et al., 2010; Martin et al., 1993; Copeland and Scott Taylor, 1995).

In this paper, we develop a framework that integrates global inequality issues in a multi-country dynamic models of transboundary pollution stock in which countries are differentiated by an endowment that yields a pollution-free flow of benefits over time and an emission technology. In our model, countries with a high level of the endowment are called richly-endowed countries while countries with low level of the endowment are called poorly-endowed countries. In addition to the non polluting flow of services, each country produces and consumes a polluting good using a given level of emission technology. We use a dynamic model because climate change is a mainly a stock problem, associated with accumulation of greenhouse gases remaining in the atmosphere for decades to centuries (Stavins, 2011; Benchekroun and Chaudhuri, 2014; Dockner and Long, 1993; Jørgensen et al., 2010; Breton et al., 2010; Martin et al., 1993; Nordhaus, 1992; Nordhaus and Yang, 1996). While the existing pollution stock is outside the direct control of policy makers, controlling the rate of emission flows can help mitigate the path of the future pollution stock. While the stock of greenhouse gases is rising, one important concern is to ensure that reducing global inequality helps to keep the future path of greenhouse-gas concentrations below a "Business-as-usual" path.

The main findings of this paper are the followings: A reduction in global inequality would have a mitigating effect on the path of the transboundary pollution stock path if the ratio rich-to-poor in the subgroup of high-tech countries is greater than the ratio rich-to-poor in the subgroup in the subgroup of low-tech countries. Another way to reinterpret this result is that a reduction in global inequality would have a mitigating effect on the path of the transboundary pollution stock path if the subgroup of high-tech countries is in average richer than the subgroup of low-tech countries. Viewed from a different perspective, the response of the transboundary pollution stock path to a reduction in global inequality results from the combined action of two opposite forces. The mitigating force can be associated with a bi-dimensional measure of both richly-endowed/high-technology countries and poorly-endowed/low-technology countries. The aggravating force can be associated with a bi-dimensional measure of both richly-endowed/low-technology countries and poorly-endowed/high-technology countries.

The rest of the paper is organized as follows. In Section 2, we present the economic model, which encompasses features of international pollution stock, emission technology, and global inequality among countries. A steady state analysis is performed to gain some preliminary intuition. Section 3 analyses the response of the transboundary pollution stock path to a reduction in global inequality in a world where richly-endowed and poorly-endowed countries use the same emission technology. Section 4 analyses the response of the transboundary pollution stock path to a reduction in inequality in world composed of two types of countries, where richly-endowed and poorly-endowed countries do not necessarily have the same emission technology level. Section 5 analyses the response of the transboundary pollution stock path to a reduction in inequality in world composed of four types of countries: richly-endowed/high-technology countries, poorly-endowed/low-technology countries, richly-endowed/low-technology countries, and poorly-endowed/high-technology countries. Section 6 offers concluding comments.

2 A model of transboundary pollution stock and global inequality

2.1 International transboundary pollution stock and a country's welfare

Consider an N-country differential game played over the time interval $[0, \infty]$, in which countries are involved in a pollution-generating economic activity. Denote by $Q_i(t)$ the industrial production of country

$i \in \{1, ..., N\}$ at time t. The production gives rise to a byproduct externality, namely (gross) emissions $e_i(t)$ at time t. For simplicity it will be assumed that one unit of production gives rise to $\frac{1}{\phi_i}$ units of pollution. In other words, production is proportional to gross emissions as follows:

$$Q_i(t) = \phi_i e_i(t). \tag{1}$$

The parameter $\phi_i > 0$ characterizes the emission technology in country i. A higher level of $\phi_i > 0$; that is a lower $\frac{1}{\phi_i}$, means that production gives rise to a lower level of emissions. Technological change, in this sense, represents the impact of exogenous forces. It rather pertains to innovation fallout that provides the country with pollution-reducing facilities (emission reducing technologies, electric car, solar energy, etc.).

The transboundary pollution stock accumulates over time according to the transition equation:

$$\dot{S}(t) = \sum_{i=1}^{N} e_i(t) - kS(t), \quad S_0 > 0 \text{ given}, \tag{2}$$

where $e_i(t)$ stands for country i's pollutant emissions at time t, $S(t)$ is the transboundary pollution stock at time t, $S_0 > 0$ is the given initial transboundary pollution stock, and the parameter $0 < k < 1$ is the natural rate at which the stock of pollution dissipates over time. Equation (2) tells us that the rate of growth of the atmospheric concentration of greenhouse gases (carbon stock) is determined by the flow of global emissions and the natural degradation of the existing stock.[1]

The damage is caused by the accumulated stock of emissions, as is the case for greenhouse gases. Each country is assumed to suffer equally from the transboundary stock of pollution.[2] the transboundary pollution stock $S(t)$ causes instantaneous damages $D(S(t)$, which is assumed to be an increasing and convex function of the transboundary pollution stock, more specifically[3]

$$D'(S(t)) > 0, \;\; and \;\; D''(S(t)) \geq 0. \tag{3}$$

Let $B(c)$ be the instantaneous gross benefit resulting from the consumption of the services provided by c. Since we have assumed a monotone relationship between industrial production and emissions, we

[1] The parameter k reflects among others the natural removal of carbon dioxide by terrestrial sinks and oceanic sinks.
[2] For this exposition, we omit regional differences in terms of vulnerability to the stock pollution problem.
[3] For this exposition, we omit regional differences in terms of vulnerability.

express the instantaneous social welfare of country i at time t for any given n-tuple emission strategy $(e_1, ..., e_N)$ as follows:

$$B\left(\gamma\phi_i e_i(t) + (1-\gamma)\left[\phi_i e_i(t) - \frac{1}{N-1}\sum_{j\neq i}\phi_j e_j(t)\right] + \pi_i\right) - D(S(t)), \quad (4)$$

where the instantaneous benefit function $u(.)$ is strictly increasing, concave, twice continuously differentiable with derivatives $B'(.)$ and $B''(.)$, and satisfies the following Inada-type assumptions

$$\lim_{c\to 0} B'(c) = \infty,\ and\ \lim_{c\to\infty} B'(c) = 0. \quad (5)$$

The parameter $0 < \gamma \leq 1$ captures a status seeking component in the behavior of a country. The concept of status-seeking is similar to the concept of interdependence in consumption discussed by Arrow et al. (2004). In this model, each country cares about not only its own absolute production but also its relative production relative to that of others. The relative production is the difference between the absolute production $\phi_i e_i(t)$ and the average production $\frac{1}{N-1}\sum_{j\neq i}\phi_j e_j(t)$. In addition to the polluting good, each country derives utility from the consumption of a non polluting good. We assume that countries are differentiated by the return, π_i, which they get on some initial endowment. The endowment yields a pollution free flow of income[4].

In terms of non-polluting endowment, two types of countries are distinguished: richly-endowed countries for whom the return on their endowment is denoted by π_2 and poorly-endowed countries for whom the return on their endowment is denoted by π_1. In addition, it is assumed that

$$\pi_2 > \pi_1, \quad (6)$$

which means that the return on the non-polluting endowment of a richly-endowed country is greater than the return on the non-polluting endowment of a poorly-endowed country. In what follows, a concept of global inequality is considered. A reduction in global inequality can be viewed as a global redistributive policy that redistributes from richly-endowed countries in favor of poorly-endowed countries.

[4]The initial endowment can be think of as a composite index that includes many indicators such as financial capacity and capabilities, natural capital, etc. This return on endowment can also be thought of as being related to the country's human capital and economic, social and political institutions.

2.2 Global inequality and redistribution from richly-endowed countries in favor of poorly-endowed countries

Let $\mathcal{R}_{\pi_1,\pi_2}$ denotes the initial distribution ("Business-as-usual" global inequality) of the non-polluting endowment across countries defined as:

$$\mathcal{R}_{\pi_1,\pi_2} = \left\{ \left(\pi_1\,,\,\frac{N_1}{N_1+N_2}\right)\,,\left(\pi_2\,,\,\frac{N_2}{N_1+N_2}\right)\right\}, \tag{7}$$

where the couple $\left(\pi_i\,,\,\frac{N_i}{N_1+N_2}\right)$ means that a proportion $\frac{N_i}{N_1+N_2}$ of countries are each endowed with π_i. To say another way, equation (7) features the initial non-polluting endowment π_1 of each of the N_1 poorly-endowed countries and the initial non-polluting endowment π_2 of each of the N_2 poorly-endowed countries richly-endowed countries. The total number of countries is $N = N_1 + N_2$.

The mean and variance of the initial global inequality distribution are given by:

$$\begin{cases} E\left(\mathcal{R}_{\pi_1,\pi_2}\right) = \frac{1}{N_1+N_2}[N_1\pi_1 + N_2\pi_2], \\ \\ \mathrm{Var}\left(\mathcal{R}_{\pi_1,\pi_2}\right) = \frac{1}{N_1+N_2}\left[N_1\Big(\pi_1 - E\left(\mathcal{R}_{\pi_1,\pi_2}\right)\Big)^2 + N_2\Big(\pi_2 - E\left(\mathcal{R}_{\pi_1,\pi_2}\right)\Big)^2\right]. \end{cases} \tag{8}$$

A reduction in global inequality in this framework is a redistribution of the non-polluting endowment from richly-endowed countries in favor of poorly-endowed countries, such as to reduce the spread of the initial distribution and to keep the average non-polluting endowment constant.

A reduction in global inequality can be represented by the following distribution:

$$\mathcal{R}_{\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)} = \left\{ \left(\widetilde{\pi}_1(\alpha)\,,\,\frac{N_1}{N_1+N_2}\right)\,,\left(\widetilde{\pi}_2(\alpha)\,,\,\frac{N_2}{N_1+N_2}\right)\right\}, \tag{9}$$

where

$$\widetilde{\pi}_1(\alpha) = \pi_1 + \frac{N_2}{N_1+N_2}(1-\sqrt{\alpha})(\pi_2-\pi_1), \tag{10}$$

$$\widetilde{\pi}_2(\alpha) = \pi_2 - \frac{N_1}{N_1+N_2}(1-\sqrt{\alpha})(\pi_2-\pi_1). \tag{11}$$

The parameter $0 < \alpha \leq 1$ captures the degree of inequality reduction between richly-endowed countries and poorly-endowed countries.[5] It is worthy of note that the endowments of richly-endowed country and

[5]The parameter $0 < \alpha \leq 1$ can be thought of as a Gini-type inequality index, where 1 indicates the "Business-as-usual" global inequality scenario and a value lower than one, $(\alpha < 1)$, indicates a reduction in global inequality.

that of poorly-endowed countries after and before and the redistribution of the non-polluting endowment from richly-endowed countries in favor of poorly-endowed countries satisfy the following relations:

$$\pi_2 > \widetilde{\pi}_2(\alpha) \geq \widetilde{\pi}_1(\alpha) > \pi_1 \quad for\ \alpha \in [0,1), \tag{12}$$

with

$$\begin{cases} \widetilde{\pi}_1(1) = \pi_1, \\ \widetilde{\pi}_2(1) = \pi_2. \end{cases} \tag{13}$$

Equation (12) tells that the redistribution is rank-preserving. Equation (13) tells us that the very particular case where $\alpha = 1$ corresponds to the initial distribution ("Business-as-usual" global inequality).

In addition, the mean and variance of the global inequality after and before redistribution of the endowment satisfy the following relations:

$$\begin{cases} E\left(\mathcal{R}_{\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)}\right) = E\left(\mathcal{R}_{\pi_1,\pi_2}\right), & \text{(14a)} \\ \\ \mathrm{Var}\left(\mathcal{R}_{\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)}\right) = \alpha \mathrm{Var}\left(\mathcal{R}_{\pi_1,\pi_2}\right) < \mathrm{Var}\left(\mathcal{R}_{\pi_1,\pi_2}\right). & \text{(14b)} \end{cases}$$

As expressed above, $\mathcal{R}_{\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)}$ expresses a redistribution of the non polluting endowment from richly-endowed countries in favor of poorly-endowed countries. Equation (14a) tells us that the global inequality after redistribution has a lower variance compared to the initial global inequality. Equation (14b) tells us that the mean of the global inequality after redistribution equals the mean of the global inequality before redistribution.[6] The parameter α captures the degree of inequality reduction among richly-endowed countries and poorly-endowed countries. The initial distribution corresponds to the very particular case where $\alpha = 1$. Latter on, we will investigate the response of the path of the transboundary pollution stock to a reduction in global inequality over the time interval $[t\,, t + dt]$.

2.3 A country's pollution control decision

Endowed with a non-polluting flow of benefits π_i, the objective of country i is to choose a pollution control strategy $\{e_i(t) \,:\, t \geq 0\}$ (or equivalently an output strategy) that maximizes the discounted stream of net benefits from consumption :

$$\max_{e_i(t);\, t\in[0,\infty[} \int_0^\infty e^{-\rho t} \left\{ B\left(\phi_i e_i(t) - \frac{(1-\gamma)}{N-1} \sum_{j\neq i} \phi_j e_j(t) + \pi_i \right) - D(S(t)) \right\} dt,$$

[6]Say another way, $\mathcal{R}_{\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)}$ is a mean-preserving spread distribution of the initial distribution $\mathcal{R}_{\pi_1,\pi_2}$.

where $\rho > 0$ is the discount rate,[7] and the maximization is subject (2) and to $e_i(t) \geq 0$.

The corresponding current value Hamiltonian is given by

$$\mathcal{H}_i(t) = B\left(\phi_i e_i(t) - \frac{(1-\gamma)}{N-1}\sum_{j\neq i}\phi_j e_j(t) + \pi_i\right) - D(S(t)) + \lambda_i(t)\left[\sum_{j=1}^{N} e_j(t) - kS(t)\right],$$

where $\lambda_i(t)$ is the shadow price associated with the transboundary pollution stock.[8]

For simplicity, we focus on interior solutions. The first order condition, co-state equation, and state equation are given by:

$$\phi_i B'\left(\phi_i e_i(t) - \frac{(1-\gamma)}{N-1}\sum_{j\neq i}\phi_j e_j(t) + \pi_i\right) + \lambda_i(t) = 0, i = 1, ..., N \tag{15}$$

$$\dot{\lambda}_i(t) - (\rho + k)\lambda_i(t) = D^{'}(S(t)), \tag{16}$$

$$\dot{S}(t) = e_i(t) + \sum_{j\neq i} e_j(t) - kS(t). \tag{17}$$

and the transversality condition is

$$\lim_{t\to\infty} e^{-\rho t}\lambda_i(t)S(t) = 0. \tag{18}$$

The shadow price on the transboundary pollution stock-accumulation equation is negative: more pollution reduces the objective function. Equation (15) is the static first-order condition for the emission (flow variable). It expresses the balance between the marginal benefit from emissions and the marginal damage from emissions, which can be shown [see equation (19)] to be the discounted value of future pollution damage generated by a marginal unit of emissions at time t. Equation (16) illustrates that the shadow price of pollution evolves at the rate determined by the marginal damage caused by a unit of pollution and the opportunity cost of reducing emissions by that unit. Equation (17) is simply a re-statement of the transboundary pollution stock dynamic in the equilibrium.

Multiplying both sides of equation (16) by $e^{-(\rho+k)t}$ and integrating both sides [taking into account the transversality condition (18)] gives the formal expression shadow price of the transboundary pollution stock as follows

$$\lambda(t) = -\int_t^{\infty} e^{-(\rho+k)(\tau-t)} D'(S(\tau))d\tau \tag{19}$$

[7]The discount rate, ρ; is assumed to be constant and identical for all countries.

[8]The Hamiltionan consists of the objective function and a constraint expressing the change in the transboundary pollution stock variable over time.

The shadow price of the transboundary pollution stock (19) is the same for all countries, i.e., $\lambda_i(t) = \lambda(t),\ i = 1, .., N$. The shadow price of the transboundary pollution stock is negative. Equation (19) tells us that the shadow price of the transboundary pollution stock is equal to the current value of all future pollution-related marginal utility loss from present time onward, discounted at the discount rate adjusted for the natural decay of the transboundary pollution stock.

2.4 Some intuition from the steady-state analysis

To get some intuition about the role played by the non-polluting endowment in influencing emission strategies, let us start with the steady state analysis. To keep matter as simple as possible, assume a world composed of two countries, a poorly-endowed country and a richly-endowed country ($N_1 = N_2 = 1$ and $\pi_2 > \pi_1$), and further assume that $\gamma = 1$. In the steady state, the stock of pollution and the shadow price of the transboundary pollution stock are constant. The first-order conditions under the steady state ($\dot{\lambda}_i = 0$ and $\dot{S} = 0$) reduce to:

$$\phi_i u'(\phi_i e_i + \pi_i) + \lambda_i = 0, \ \ i = 1, 2, \tag{20}$$

$$\lambda_i = -\frac{D'(S)}{\rho + k}, \tag{21}$$

$$S = \frac{e_1 + e_2}{k}. \tag{22}$$

Substituting (21) and (22) into (20) leads to the following system of equations:

$$\begin{cases} \phi_1 B'(\phi_1 e_1 + \pi_1) = \dfrac{1}{\rho + k} D'(\dfrac{e_1 + e_2}{k}), & \text{(23a)} \\ \\ \phi_2 B'(\phi_2 e_2 + \pi_2) = \dfrac{1}{\rho + k} D'(\dfrac{e_1 + e_2}{k}). & \text{(23b)} \end{cases}$$

The steady state equilibrium is hard to solve analytically in a general case. However, due to the concavity assumption on the benefit function and the convexity assumption on the damage function, there exists a unique steady state equilibrium.

It is often of interest to determine how a change in underlying conditions affects the best response function. Let us assume that the emission e_2 is given. The marginal effect of an increase in the non-polluting endowment on the best response function $e_1(e_2)$ of the poorly-endowed country, obtained by differentiating equation (23a), is obtained as :

$$\frac{de_1(e_2)}{d\pi_1} = \frac{\underbrace{-\phi_1 B''(\phi_1 e_1 + \pi_1)}_{>0}}{\underbrace{\phi_1^2 B''(\phi_1 e_1 + \pi_1) - \frac{D'(\frac{e_1+e_2}{k})}{k(\rho+k)}}_{<0}} < 0 \tag{24}$$

The negative sign of the derivative in equation (24) tells us that increasing the non-polluting endowment of the poorly-endowed country, while keeping all other factors constant, will result in a lower (less aggressive) emission response strategy.

Equalizing the right-hand sides of equations (23a) and (23b) shows that, in the steady-sate equilibrium, the emissions strategies of both countries satisfy the following relationship:

$$\phi_1 B'(\phi_1 e_1 + \pi_1) = \phi_2 B'(\phi_2 e_2 + \pi_2). \tag{25}$$

If it is further assumed that the emissiontechnologies are identical for both countries; that is $\phi_2 = \phi_1 = \phi$, the gap between the emission the of poorly endowed-country and that of the richly-endowed country in the steady-state equilibrium is given by:

$$e_1 - e_2 = \frac{\pi_2 - \pi_1}{\phi} > 0. \tag{26}$$

Equation (26) tells us that, in the steady-state equilibrium, the emission of the poorly-endowed country is always higher than that of the richly-endowed country. Reducing the gap between the non-polluting endowment of the poorly-endowed country and that of the richly endowed countries will reduce the gap in their emission strategies in the steady state equilibrium.

The steady-state analysis provides some intuition about the crucial role that the non-pollution endowment will play in influencing optimal emissions strategies. However, not having an analytical solution hinders the investigation of the response of the global pollution flow to a reduction in global inequality. It is also worth mentioning that the climate change is mainly a stock problem, not a flow problem. Damages are caused by stock externalities (stock of greenhouse gases), and there are concerns that the world stock of greenhouse gases will continue to rapidly increase in the atmosphere over the next decades. This suggests that a dynamic analysis is more appropriate to explore the response of the stock pollution path to a reduction in global inequality. The next sections will follow a dynamic analysis approach.

3 Response of the path of pollution stock to a reduction in global inequality: case of identical emission technology among countries, $\phi_1 = \phi_2 = 1$

3.1 Equilibrium emissions

In this section, we consider a world composed of N_1 poorly-endowed countries and N_2 richly-endowed countries, $N = N_1 + N_2$. For the sake of simplicity, assume that $\phi_1 = \phi_2 = 1$. Therefore, from equation (15) and noting that $\lambda_1(t) = \lambda_2(t) = \lambda(t)$, the following system is obtained.

$$\begin{cases} \left[1-(1-\gamma)\frac{N_1-1}{N-1}\right] e_1(t) - (1-\gamma)\frac{N_2}{N-1} e_2(t) = B'^{-1}(-\lambda(t)) - \pi_1, \\ \\ -(1-\gamma)\frac{N_1}{N-1} e_1(t) + \left[1-(1-\gamma)\frac{N_2-1}{N-1}\right] e_2(t) = B'^{-1}(-\lambda(t)) - \pi_2, \end{cases} \tag{27}$$

where $N = N_1 + N_2$.

Solving the system (27) gives the equilibrium emissions of both countries as follows.

$$\begin{cases} e_1\left(-\lambda(t), \pi_1, \pi_2, N_1, N_2\right) = \frac{\left[B'^{-1}(-\lambda(t))-\pi_1\right]\left[1-(1-\gamma)\frac{N_2-1}{N-1}\right]+(1-\gamma)\frac{N_2}{N-1}\left[B'^{-1}(-\lambda(t))-\pi_2\right]}{D}, \\ \\ e_2\left(-\lambda(t), \pi_1, \pi_2, N_1, N_2\right) = \frac{\left[B'^{-1}(-\lambda(t))-\pi_2\right]\left[1-(1-\gamma)\frac{N_1-1}{N-1}\right]+(1-\gamma)\frac{N_1}{N-1}\left[B'^{-1}(-\lambda(t))-\pi_1\right]}{D}, \end{cases} \tag{28}$$

where

$$\begin{aligned} D &= \left[1-(1-\gamma)\frac{N_1-1}{N-1}\right]\left[1-(1-\gamma)\frac{N_2-1}{N-1}\right] - \left(\frac{1-\gamma}{N-1}\right)^2 N_1 N_2 \\ &= \gamma\left[\frac{N-\gamma}{N-1}\right] > 0. \end{aligned} \tag{29}$$

Equation (28) the optimal emissions strategies in closed-form expressions in terms of the shadow price of the transboundary pollution stock.

From equation (28), it can be shown that

$$e_1\left(\lambda(t), \pi_1, \pi_2, N_1, N_2\right) - e_2\left(\lambda(t), \pi_1, \pi_2, N_1, N_2\right) = \frac{\gamma}{D}(\pi_2 - \pi_1) > 0. \tag{30}$$

So, with identical level of emission technology, poorly-endowed countries are more polluting than richly-endowed countries in a asymmetric world composed of both types of countries. A somewhat similar result appears in equation (26) from the the steady state analysis.

3.2 Neutral response of the path of the transboundary pollution stock to a reduction in global inequality

In this section we will compare the path of the transboundary pollution stock under a reduction in global inequality that occurs over the time interval $[t\,,\,t+dt]$ with the path of the transboundary pollution stock that would prevail under a "Business-as-usual" global inequality scenario.

Let us assume that over the time interval $[t\,,\,t+dt]$, a reduction global inequality, represented by the distribution $\mathcal{R}_{\widetilde{\pi}_1(\alpha)\,,\widetilde{\pi}_2(\alpha)}$, discussed in Section (2.2), occurs. In order words, over the time interval $[t\,,\,t+dt]$, the return on the non-polluting endowment becomes

$$\widetilde{\pi}_1(\alpha) = \pi_1 + \frac{N_2}{N_1+N_2}(1-\sqrt{\alpha})(\pi_2-\pi_1), \quad (31)$$

$$\widetilde{\pi}_2(\alpha) = \pi_2 - \frac{N_1}{N_1+N_2}(1-\sqrt{\alpha})(\pi_2-\pi_1), \quad (32)$$

where the parameter $0<\alpha\leq 1$ characterizes the degree of inequality reduction, with the case $\alpha=1$ representing the "business-as-usual global inequality" scenario.

With a reduction in global inequality over the time interval $[t\,,t+dt]$, the rate of change of the transboundary pollution stock is given by

$$\dot{S}\left(\lambda(t),\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)\right) = E\left(\lambda(t),\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)\right) - kS(t), \quad (33)$$

where $E\left(\lambda(t),\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)\right) = N_1 e_1\left(\lambda(t),\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)\right) + N_2 e_2\left(\lambda(t),\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)\right)$ is the global flow of emissions under the reduction in global inequality and $S(t)$ is the existing stock of pollution at time t.

Under the "Business-as-usual" global inequality scenario over the interval $[t\,,t+dt]$, the rate of change of the transboundary pollution stock is given by

$$\dot{S}\left(\lambda(t),\pi_1,\pi_2\right) = E\left(\lambda(t),\pi_1,\pi_2\right) - kS(t) > 0, \quad (34)$$

where $E\left(\lambda(t),\pi_1,\pi_2\right) = N_1 e_1\left(\lambda(t),\pi_1,\pi_2\right) + N_2 e_2\left(\lambda(t),\pi_1,\pi_2\right)$ is the global flow of emissions under the 'Business-as-usual" global inequality scenario and $S(t)$ is the existing stock of pollution at time t.

From equations (33) and (34), it follows that

$$\dot{S}\left(\lambda(t),\pi_1(\alpha),\pi_2(\alpha)\right) - \dot{S}\left(\lambda(t),\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)\right) = E\left(\lambda(t),\pi_1,\pi_2,N_1,N_2\right) - E\left(\lambda(t),\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)\right). \quad (35)$$

Using equations (35), (28), (10) and (11), it follows that

$$\begin{aligned}\dot{S}\left(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)\right) - \dot{S}\left(\lambda(t), \widetilde{\pi}_1(\alpha), \widetilde{\pi}_2(\alpha)\right) &= \frac{1}{D}\left(\frac{N-\gamma}{N-1}\right) N_1(\widetilde{\pi}_1(\alpha) - \pi_1) + N_2(\widetilde{\pi}(\alpha) - \pi_1), \\ &= \frac{1}{D}\left(\frac{N-\gamma}{N-1}\right) N_1 N_2 (1-\sqrt{\alpha})(\pi_2 - \pi_1 + \pi_1 - \pi_2), \\ &= 0. \end{aligned} \tag{36}$$

With a reduction in global inequality, the rate of change of the transboundary pollution stock over the time interval $[t, t+dt]$ is given by

$$\dot{S}\left(\lambda(t), \widetilde{\pi}_1(\alpha), \widetilde{\pi}_2(\alpha)\right) = \frac{S_\alpha(t+dt) - S(t)}{dt}, \tag{37}$$

where $S_\alpha(t+dt)$ is the future stock of pollution at time $t+dt$ that results from a reduction in global inequality.

Under the "Business-as-usual" global inequality scenario, the rate of change of the transboundary pollution stock over the time interval $[t, t+dt]$ is given by

$$\dot{S}\left(\lambda(t), \pi_1, \pi_2\right) = \frac{S_1(t+dt) - S(t)}{dt}, \tag{38}$$

where $S_1(t+dt)$ is the future stock of pollution at time $t+dt$ that results from the 'Business-as-usual" global inequality scenario.

From equations (36), (37), (38), (31), and (32), it follows that the path of the pollution stock under a reduction in global inequality is similar to the path of the transboundary pollution stock under ''Business-as-usual" global inequality scenario, as shown in the following equation:

$$\underbrace{S_1(t+dt) - S_\alpha(t+dt)}_{Net\,Response\,of\,the\,Pollution\,\,Stock} = 0, \tag{39}$$

where $S_1(t+dt)$ is the transboundary pollution stock at time $t+dt$ under the "Business-as-usual" global inequality scenario and $S_\alpha(t+dt)$ is the transboundary pollution stock at time $t+dt$ that results from a reduction in global inequality.

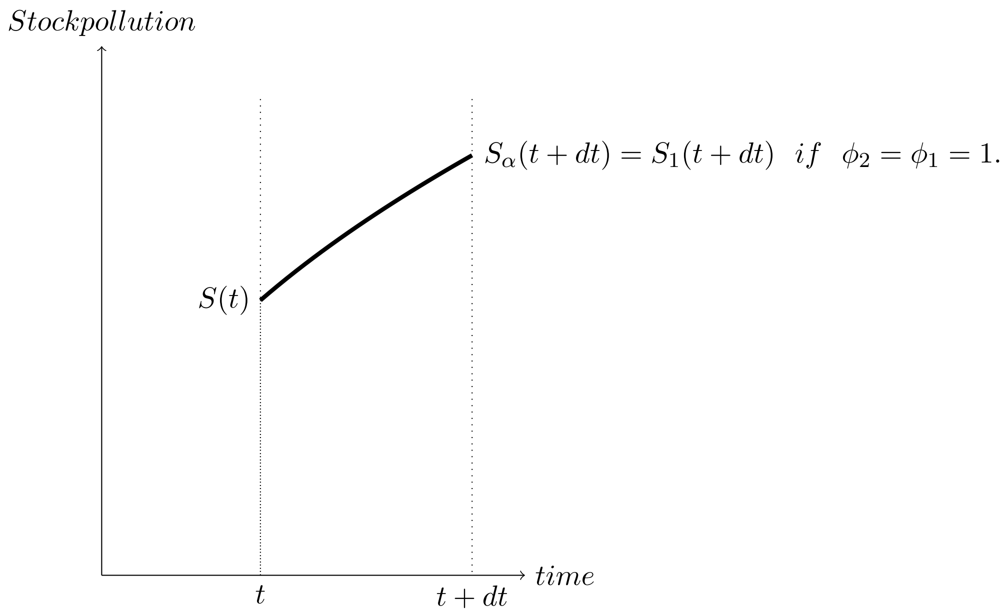

Figure 1: Neutral response of the path of the transboundary pollution stock to a reduction in global inequality.

As illustrated in Figure 1, equation (39) means that a reduction in global inequality will not shift the path of the transboundary pollution stock away from the path that would prevail under the "Business-as-usual" global inequality scenario. When rich countries and poor countries have the same technology, a redistribution form richly-endowed countries in favor of the poorly-endowed countries will decrease the emissions of poorly-endowed countries; however, this decrease in emissions is equally compensated by the increase in emissions from richly-endowed countries. As a result both effects annihilate each other leading to a neutral response of the path of the international transboundary pollution stock to a reduction in global inequality over the time interval $[t, t+dt]$.

4 Response of the path of pollution stock to a reduction in global inequality in a world composed of two types of countries: poorly-endowed countries countries and richly-endowed countries, with $\phi_1 \neq \phi_2$ or $\phi_1 \neq \phi_2$

In this section let us assume that ϕ_2 is the level of technology owned by each richly-endowed country and ϕ_1 is the level of technology owned by each poorly-endowed country. In addition, assume a world composed of N_2 richly-endowed countries and N_1 poorly-endowed countries ($N = N_1 + N_2$). The response of the path of the polluting stock to a reduction in global inequality will be analyzed in the following cases: $\phi_2 < \phi_1$; $\phi_2 = \phi_1$; and $\phi_2 > \phi_1$.

The system of equations for the equilibrium emissions is obtained from the first oder conditions (15) as follows:

$$\left\{\begin{array}{l}\phi_1\left[1-(1-\gamma)\frac{N_1-1}{N-1}\right]e_1(t)-\phi_2(1-\gamma)\frac{N_2}{N-1}e_2(t)=B'^{-1}(\frac{\lambda(t)}{\phi_1})-\pi_1,\\ \\ -\phi_1(1-\gamma)\frac{N_1}{N-1}e_1(t)+\phi_2\left[1-(1-\gamma)\frac{N_2-1}{N-1}\right]e_2(t)=B'^{-1}(\frac{\lambda(t)}{\phi_2})-\pi_2.\end{array}\right. \quad (40)$$

After solving the system (40), the equilibrium emissions of both type of countries are as follows.

$$\left\{\begin{array}{l}e_1\left(\lambda(t),\pi_1,\pi_2,N_1,N_2\right)=\frac{\left[B'^{-1}(\frac{-\lambda(t)}{\phi_1})-\pi_1\right]\left[1-(1-\gamma)\frac{N_2-1}{N-1}\right]+(1-\gamma)\frac{N_2}{N-1}\left[B'^{-1}(\frac{-\lambda(t)}{\phi_2})-\pi_2\right]}{\phi_1\,D},\\ \\ e_2\left(\lambda(t),\pi_1,\pi_2,N_1,N_2\right)=\frac{\left[B'^{-1}(\frac{-\lambda(t)}{\phi_2})-\pi_2\right]\left[1-(1-\gamma)\frac{N_1-1}{N-1}\right]+(1-\gamma)\frac{N_1}{N-1}\left[B'^{-1}(\frac{-\lambda(t)}{\phi_1})-\pi_1\right]}{\phi_2\,D},\end{array}\right. \quad (41)$$

where

$$D \;=\; \gamma\left[\frac{N-\gamma}{N-1}\right]>0.$$

Let us assume that over the time interval $[t\,,\,t+dt]$, a reduction in global inequality occurs. In order words over the time interval $[t\,,\,t+dt]$, the return on the non-polluting endowment becomes

$$\widetilde{\pi}_1(\alpha) \;=\; \pi_1+\frac{N_2}{N_1+N_2}(1-\sqrt{\alpha})(\pi_2-\pi_1), \quad (42)$$

$$\widetilde{\pi}_2(\alpha) \;=\; \pi_2-\frac{N_1}{N_1+N_2}(1-\sqrt{\alpha})(\pi_2-\pi_1), \quad (43)$$

where the parameter $0<\alpha\leq 1$ characterizes the degree of inequality reduction, with the case $\alpha=1$ representing the "Business-as-usual" global inequality scenario [see Section (2.2)].

Using equations (41), (35), (37), (38), (42) and (43), it can be shown that the net response between the path of the pollution stock under a reduction in global equality and the path that would prevail under the "Business-as-usual" global inequality' scenario is given by:

$$\underbrace{S_1(t+dt)-S_\alpha(t+dt)}_{Net\,Response\,of\,the\,Pollution\,\,Stock} \;=\; \left[\frac{(N-1)(1-\sqrt{\alpha})(\pi_2-\pi_1)}{N(N-\gamma)}N_1N_2\left(\frac{1}{\phi_1}-\frac{1}{\phi_2}\right),\right]dt, \quad (44)$$

where $S_1(t+dt)$ is the transboundary pollution stock at time $t+dt$ under the "Business-as-usual" global inequality scenario and $S_\alpha(t+dt)$ is the transboundary pollution stock at time $t+dt$ that results from a reduction in global inequality.

As implied by the term $\frac{1}{\phi_1}-\frac{1}{\phi_2}$ in equation (44), the gap between the emission technology of the N_2 richly-endowed countries and that of the N_1 poorly-endowed countries is crucial in determining the response of the transboundary pollution stock path to a reduction in global inequality as follows.

$$\left\{\begin{array}{lll}S_\alpha(t+dt)>S_1(t+dt) & \text{if} & \phi_2<\phi_1, \quad (45a)\\ S_\alpha(t+dt)=S_1(t+dt) & \text{if} & \phi_2=\phi_1, \quad (45b)\\ S_\alpha(t+dt)<S_1(t+dt) & \text{if} & \phi_2>\phi_1, \quad (45c)\end{array}\right.$$

where $S_1(t+dt)$ is the transboundary pollution stock at time $t+dt$ under the "Business-as-usual" global inequality scenario and $S_\alpha(t+dt)$ is the transboundary pollution stock at time $t+dt$ that results from a reduction in global inequality. The degree of inequality parameter $0 < \alpha \leq 1$ and the status parameter $\gamma > 0$ in equation (44) act as multipliers of the magnitude of the net response of the transboundary pollution stock path.

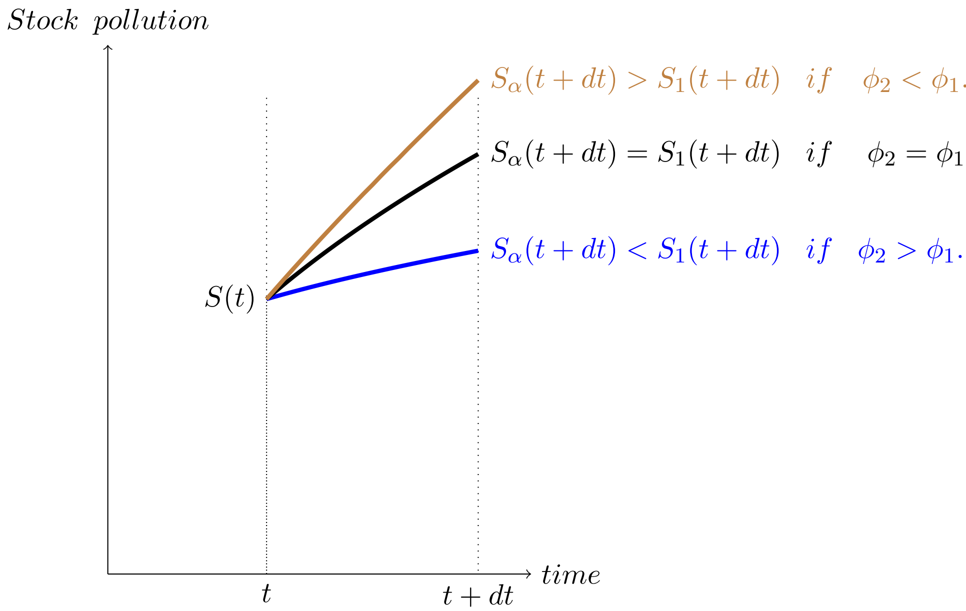

Figure 2: Response of the transboundary pollution stock path to a reduction in global inequality: Gap in technology explanation

Figure 2 depicts plausible paths of the response of the transboundary pollution stock path can take after a reduction in global inequality from a gap in technology approach. In a world composed of N_2 richly-endowed countries that have the high emission technology and N_1 poorly-endowed countries that have the low emission technology ($\phi_2 > \phi_1$), a reduction in global inequality will mitigate the path of the transboundary pollution stock by curbing the transboundary pollution stock below the path that would prevail under the "Business-as-usual" global inequality scenario. In contrast, in a world composed of N_2 richly-endowed countries that have the low technology and N_1 poorly-endowed countries that have the high technology ($\phi_2 < \phi_1$), a reduction in global inequality will aggravate the transboundary pollution stock problem by elevating the path of the international pollution stock above the path that would prevail under the "Business-as-usual" global inequality scenario. Note also that in a world composed of the N_2 richly-endowed countries and N_1 poorly-endowed countries that all have all the same level of emission technology ($\phi_2 = \phi_1$), a reduction in global inequality will not shit the path of the international pollution stock away from the path that would prevail under the "Business-as-usual" global inequality scenario. In the very particular case where $\phi_2 = \phi_1 = 1$, this reduces to the neutrality result obtained in the preceding Section [See equation (39)].

5 How does the path the transboundary pollution stock respond to a reduction global inequality in a world in which coexist richly-endowed/high-technology countries, poorly-endowed/low-technology countries, richly-endowed/low-technology countries, and poorly-endowed/high-technology countries

Our analysis so far has assumed three situations in a world composed of two types of countries: a world composed of richly-endowed countries that have the high technology and poorly-endowed countries that have the low technology; a world composed of richly-endowed countries that have the low technology and poorly-endowed countries that have the high technology; and a world composed of richly-endowed countries and poorly-endowed countries that have the same technology.

Let us push the analysis further by considering a world where four types of countries coexist: the group of richly-endowed countries that have a high technology, the group of richly-endowed countries that have a low technology, the group of poorly-endowed countries that have a high technology, and the group of poorly-endowed countries that have a low technology. In other words, countries are sorted by both technology and endowment. In what follows it is assumed that

$$\phi_2 \geq \phi_1. \tag{46}$$

Let us denote by e_{ij}, the emission of a country with emission technology ϕ_i and return on endowment π_j, with $i, j \in \{1, 2\}$. Let $\mathcal{R}_{\pi_1,\pi_2}$ denotes the initial distribution (''Business-as-usual" global inequality) of the non-polluting endowment across countries defined as

$$\mathcal{R}_{\pi_1,\pi_2} = \left\{ \left(\pi_1\,, \frac{N_{11}}{N}\right)\,, \left(\pi_1\,, \frac{N_{21}}{N}\right)\,, \left(\pi_2\,, \frac{N_{12}}{N}\right)\,, \left(\pi_2\,, \frac{N_{22}}{N}\right) \right\}, \tag{47}$$

where $N = N_{11} + N_{12} + N_{21} + N_{22}$, and the couple $\left(\pi_j\,, \frac{N_{ij}}{N}\right)$ means that a proportion $\frac{N_{ij}}{N}$ have the technology level ϕ_i and the endowment π_j.

The equilibrium emissions strategies are computed [seeAppendix] as follows:

$$e_{11}(t) = \frac{1}{\phi_1\gamma(N-\gamma)} \Big\{ [N-\gamma-(1-\gamma)(N_{21}+N_{22})] \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_1}\right) - \pi_1 \right]$$
$$+(1-\gamma)(N_{21}+N_{22}) \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_2}\right) - \pi_2 \right] + (1-\gamma)(N_{21}-N_{12})(\pi_2-\pi_1) \Big\}, \tag{48}$$

$$e_{22}(t) = \frac{1}{\phi_2\gamma(N-\gamma)} \Big\{ [N-\gamma-(1-\gamma)(N_{11}+N_{12})] \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_2}\right) - \pi_2 \right]$$
$$+(1-\gamma)(N_{11}+N_{12}) \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_1}\right) - \pi_1 \right] + (1-\gamma)(N_{21}-N_{12})(\pi_2-\pi_1) \Big\}, \tag{49}$$

$$e_{12}(t) = \frac{1}{\phi_1\gamma(N-\gamma)} \Big\{ [N-\gamma-(1-\gamma)(N_{21}+N_{22})] \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_1}\right) - \pi_1 \right]$$
$$+(1-\gamma)(N_{21}+N_{22}) \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_2}\right) - \pi_2 \right] + [(1-\gamma)(N_{21}-N_{12}) - \gamma(N-1)]\,(\pi_2-\pi_1) \Big\}, \tag{50}$$

$$e_{21}(t) = \frac{1}{\phi_2\gamma(N-\gamma)} \left\{ [N-\gamma-(1-\gamma)(N_{11}+N_{12})] \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_2}\right) - \pi_2 \right] \right.$$
$$\left. +(1-\gamma)(N_{11}+N_{12}) \left[B'^{-1}\left(\frac{-\lambda(t)}{\phi_1}\right) - \pi_1 \right] + [(1-\gamma)(N_{21}-N_{12})+\gamma(N-1)] \right\}. \quad (51)$$

It is worth noting [from see equations (72) and (73) in the Appendix] that

$$e_{11}(t) > e_{12}(t), \quad (52)$$
$$and$$
$$e_{21}(t) > e_{22}(t). \quad (53)$$

Equation (52) tells us that poorly-endowed countries that have a low technology are more polluting than richly-endowed countries that have a low technology. Equation (53) tells us that poorly-endowed countries that have a high technology are more polluting than richly-endowed countries that have a high technology.

From 72) and (73), it can also be shown that

$$\underbrace{e_{11}(t)-e_{12}(t)}_{Gap\, in\, emissions\, among\, poorly-endowed\, countries} = \frac{\phi_2}{\phi_1}(e_{21}(t)-e_{22}(t)) > \underbrace{e_{21}(t)-e_{22}(t)}_{Gap\, in\, emissions\, among\, richly-endowed\, countries} \quad (54)$$

Equation (54) tells that the gap in emission between low-tech and high-tech countries in the poorly-endowed subgroup is greater than the gap in emission between low-tech and high-tech countries in the richly-endowed subgroup. This gap in emission can be seen as a very particular indicator of group-efficiency that shows that the group of richly-endowed countries is not necessarily always more environmentally friendly than the group of poorly endowed countries. An intuitive explanation of this result may be extended to the green paradox literature, which suggests the possibility that environmental friendly factors may end-up having more negative environmental consequences [see for instance Benchekroun and Chaudhuri (2014); Hoel (1991); Long (2015)]; or, as in the case of the gap in emission, less positive environmental consequences.

A reduction in global inequality in this framework is a redistribution of the non-polluting endowment from richly-endowed countries in favor of poorly-endowed countries, such as to reduce the spread of the initial distribution and to keep the average non-polluting endowment constant. Let us assume that over the time interval $[t\,,\,t+dt]$, the following reduction global inequality occurs.

$$\mathcal{R}_{\widetilde{\pi}_1(\alpha),\widetilde{\pi}_2(\alpha)} = \left\{ \left(\widetilde{\pi}_1(\alpha)\,,\frac{N_{11}}{N}\right), \left(\widetilde{\pi}_1(\alpha)\,,\frac{N_{21}}{N}\right), \left(\widetilde{\pi}_2(\alpha)\,,\frac{N_{12}}{N}\right), \left(\widetilde{\pi}_2(\alpha)\,,\frac{N_{22}}{N}\right) \right\}, \quad (55)$$

where

$$\widetilde{\pi}_1(\alpha) = \pi_1 + \frac{N_{11}+N_{21}}{N}(1-\sqrt{\alpha})(\pi_2-\pi_1), \quad (56)$$
$$\widetilde{\pi}_2(\alpha) = \pi_2 - \frac{N_{12}+N_{22}}{N}(1-\sqrt{\alpha})(\pi_2-\pi_1), \quad (57)$$

where the parameter $0 < \alpha \leq 1$ characterizes the degree of global inequality reduction, with the case $\alpha = 1$ representing the "business-as-usual global inequality" scenario [similarly to Section (2.2)] .

Using equations (48)-(51), (35), (37), (38), (56) and (57), it can be shown that the net response between the path of the pollution stock under a reduction in equality and the path that would prevail under the "Business-as-usual" global inequality scenario is given by:

$$\underbrace{S_1(t+dt) - S_\alpha(t+dt)}_{Net\, Response\, of\, the\, Pollution\, Stock} = \left[\frac{(N-1)(1-\sqrt{\alpha})(\pi_2-\pi_1)}{N(N-\gamma)}(N_{11}N_{22} - N_{12}N_{21})\left(\frac{1}{\phi_1} - \frac{1}{\phi_2}\right)\right] dt, \tag{58}$$

where $S_1(t+dt)$ is the transboundary pollution stock at time $t+dt$ under the "business-as-usual" global inequality scenario and $S_\alpha(t+dt)$ is the transboundary pollution stock at time $t+dt$ that results from a reduction in global inequality. The degree of inequality parameter $0 < \alpha \leq 1$ and the status parameter $\gamma > 0$ in equation (58) act as multipliers of the magnitude of the net response of the transboundary pollution stock.

Having assumed that $\phi_2 \geq \phi_1$, it it can be seen through the term $N_{11}N_{22} - N_{12}N_{21}$ in equation (58) that the structure of the composition of countries in terms of both technology and endowment is crucial in determining the response of the transboundary pollution stock to a reduction in global inequality as follows.

$$\begin{cases} S_\alpha(t+dt) > S_1(t+dt) \quad \text{if} \quad N_{11}N_{22} - N_{12}N_{21} < 0, & \text{(59a)} \\ S_\alpha(t+dt) = S_1(t+dt) \quad \text{if} \quad N_{11}N_{22} - N_{12}N_{21} = 0, & \text{(59b)} \\ S_\alpha(t+dt) < S_1(t+dt) \quad \text{if} \quad N_{11}N_{22} - N_{12}N_{21} > 0, & \text{(59c)} \end{cases}$$

where $S_1(t+dt)$ is the transboundary pollution stock at time $t+dt$ under the "Business-as-usual" global inequality scenario and $S_\alpha(t+dt)$ is the transboundary pollution stock at time $t+dt$ that results from a reduction in global inequality. In other to give a first interpretation to equation (59), it is worth noting that the condition $N_{11}N_{22} - N_{12}N_{21} > 0$ can be rewritten [assuming $N_{21} \neq 0$ and $N_{11} \neq 0$] as

$$\underbrace{\frac{N_{22}}{N_{21}}}_{Ratio\, Rich-to-Poor\, in\, the\, subgroup\, of\, high-tech\, countries} > \underbrace{\frac{N_{12}}{N_{11}}}_{Ratio\, rich-to-poor\, in\, the\, subgroup\, of\, low-tech\, countries} . \tag{60}$$

Therefore, equations (59a), (59b), and (59c) can be rewritten as:

$$\begin{cases} S_\alpha(t+dt) > S_1(t+dt) \quad \text{if} \quad \dfrac{N_{22}}{N_{21}} < \dfrac{N_{12}}{N_{11}}, & \text{(61a)} \\ S_\alpha(t+dt) = S_1(t+dt) \quad \text{if} \quad \dfrac{N_{22}}{N_{21}} = \dfrac{N_{12}}{N_{11}}, & \text{(61b)} \\ S_\alpha(t+dt) < S_1(t+dt) \quad \text{if} \quad \dfrac{N_{22}}{N_{21}} > \dfrac{N_{12}}{N_{11}}, & \text{(61c)} \end{cases}$$

Equation (61c) tells us that a reduction in global inequality would have a mitigating effect on the path of the transboundary pollution stock path if the subgroup of high-tech countries contains relatively more rich countries than the subgroup of low-tech countries does. Equation (61b) tells us that if the relative number of richly endowed countries in the subgroup of high-tech countries is equal the relative number of richly-endowed countries in the subgroup of low-tech countries, a reduction in global inequality would have a neutral effect on the path of the transboundary pollution stock. This neutrality result parallels the paper by Copeland and Scott Taylor (1995) in the trade literature who found that international income transfers may not affect world pollution. Along the same lines, this is also reminiscent of the neutrality result found by Warr (1983) in the public finance literature. Another take away from analyzing equation (61b) is that if the technology gap between the subgroup of hich-tech countries and the sub-group of low-tech countries is very large, the response of the path of the future pollution stock would be neutral to a reduction in global inequality if both subgroups have the same relative number of rich countries (ratio rich-to-poor countries). Equation (61a) tells us that a reduction in global inequality would have an aggravating effect on the path of the transboundary pollution stock path if the subgroup of high-tech countries contains relatively fewer rich countries than the subgroup of low-tech countries does.

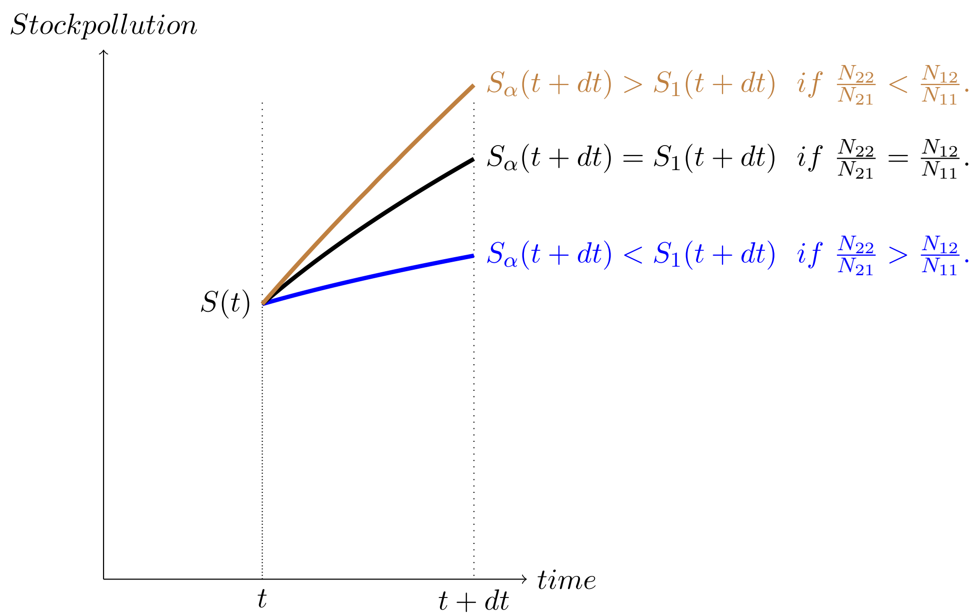

Figure 3: Response of the transboundary pollution stock path to a reduction in global inequality: Ratio rich-poor explanation

Figure (3) depicts plausible responses of the transboundary pollution stock path to a reduction in global inequality as it relates to the relative number of rich countries within the subgroup of high-tech countries.

Let us assume that the condition $\frac{N_{22}}{N_{21}} > \frac{N_{12}}{N_{11}}$ is satisfied. Equation (61c) suggests that a lower gap in emission technology between the subgroup of high-tech countries and the subgroup of low-tech countries will lead to a smaller net mitigating response of the pollution stock path. If it is further assumed that the emission technology are identical, $\phi_1 = \phi_2$, then reducing global inequality would have a neutral effect on the path of the future pollution stock. This result can be connected to Benchekroun and Chaudhuri (2014) who have shown that within a transboundary non-cooperative pollution game, a technology transfer may not result in a Pareto improvement.

An alternative way of analyzing the response of the transboundary pollution stock path to a reduction in global inequality is by noticing that equation (58) can be rewritten as

$$\underbrace{S_1(t+dt) - S_\alpha(t+dt)}_{Net\ Response\ of\ the\ Pollution\ Stock} = \left[\frac{(N-1)(N_{11}+N_{12})(N_{21}+N_{22})(1-\sqrt{\alpha})(\pi_2-\pi_1)}{N(N-\gamma)[\widetilde{\pi}_2(\alpha)-\widetilde{\pi}_1(\alpha)]}(\bar{\Pi}_{\phi_2}(\alpha)-\bar{\Pi}_{\phi_1}(\alpha))\left(\frac{1}{\phi_1}-\frac{1}{\phi_2}\right)\right]dt, \tag{62}$$

where $\bar{\Pi}_{\phi_2}(\alpha) = \frac{N_{21}\widetilde{\pi}_1(\alpha)+N_{22}\widetilde{\pi}_2(\alpha)}{N_{21}+N_{22}}$ is the average return on the endowment of the subgroup of high-technology countries after reduction in global inequality and $\bar{\Pi}_{\phi_1}(\alpha) = \frac{N_{11}\widetilde{\pi}_1(\alpha)+N_{12}\widetilde{\pi}_2(\alpha)}{N_{11}+N_{12}}$ is the average return on the endowment of the subgroup of low-technology countries after reduction in global inequality.[9]

From equation (62), it can be seen that the gap in the average return on endowment between the subgroup of high tech counties and that of low-tech countries, $\bar{\Pi}_{\phi_2}(\alpha) - \bar{\Pi}_{\phi_1}(\alpha)$, is a predictor of the response of the transboundary pollution stock to a reduction in global inequality as follows.

$$\begin{cases} S_\alpha(t+dt) > S_1(t+dt) \quad \text{if} \quad \bar{\Pi}_{\phi_2}(\alpha) < \bar{\Pi}_{\phi_1}(\alpha), & \text{(63a)} \\ S_1(t+dt) = S_\alpha(t+dt) \quad \text{if} \quad \bar{\Pi}_{\phi_2}(\alpha) = \bar{\Pi}_{\phi_1}(\alpha), & \text{(63b)} \\ S_\alpha(t+dt) < S_1(t+dt) \quad \text{if} \quad \bar{\Pi}_{\phi_2}(\alpha) > \bar{\Pi}_{\phi_1}(\alpha), & \text{(63c)} \end{cases}$$

where $S_1(t+dt)$ is the transboundary pollution stock at time $t+dt$ under the "Business-as-usual" global inequality scenario and $S_\alpha(t+dt)$ is the transboundary pollution stock at time $t+dt$ that results from a reduction in global inequality. A reduction in global inequality will shift the path of the transboundary pollution stock below that of a "business-as-usual global inequality" scenario if the average return on endowment in the subgroup of high-technology countries after reduction in global inequality is greater than the average return on endowment in the subgroup of low-technology countries after reduction in global inequality.

[9] It is worth noting that $\bar{\Pi}_{\phi_1}(\alpha) - \bar{\Pi}_{\phi_2}(\alpha) = -[N_{11}N_{22} - N_{11}N_{22}]\frac{[\widetilde{\pi}_2(\alpha)-\widetilde{\pi}_1(\alpha)]}{(N_{11}+N_{12})(N_{21}+N_{22})}$

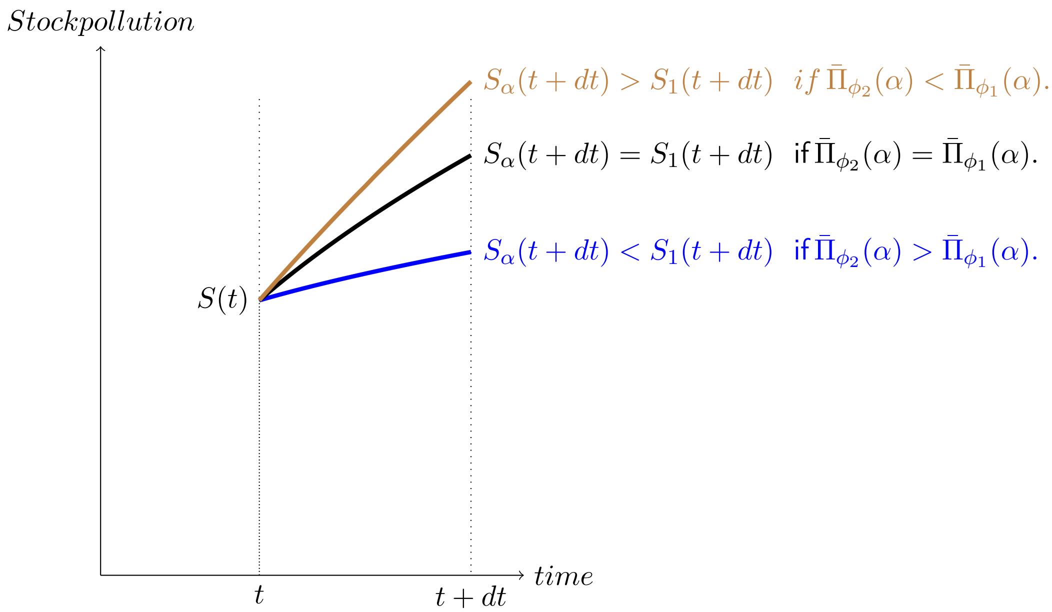

Figure 4: Response of the transboundary pollution stock path to a reduction in global inequality: Average endowment explanation

Figure (4) depicts plausible responses of the transboundary pollution stock path of the transboundary pollution stock path to a reduction in global inequality as it relates to the average endowment of high-tech countries.

Finally let us provide a complementary interpretation by introducing the matrix of world classification of countries in terms of both technology and endowment levels, that is:

$$\begin{array}{c c} & \begin{array}{cc} poor & rich \end{array} \\ \begin{array}{c} low\, tech \\ high\, tech \end{array} & \begin{pmatrix} N_{1,1} & N_{1,2} \\ N_{2,1} & N_{2,2} \end{pmatrix}. \end{array}$$

It is worth noting that the determinant of the matrix of world classification of countries is exactly the expression $N_{11}N_{22} - N_{12}N_{21}$ that appears in equation (58). This expression can be rewritten as follows:

$$N_{11}N_{22} - N_{12}N_{21} = \det \begin{pmatrix} N_{1,1} & N_{12} \\ N_{21} & N_{2,2} \end{pmatrix} = det \underbrace{\begin{pmatrix} N_{1,1} & 0 \\ 0 & N_{2,2} \end{pmatrix}}_{N_{11}N_{22}} + \det \underbrace{\begin{pmatrix} 0 & N_{1,2} \\ N_{2,1} & 0 \end{pmatrix}}_{-N_{12}N_{21}}. \quad (64)$$

To express equation (64) geometrically, it is useful to define the two concepts that are its critical components; $N_{11}N_{22}$ and $N_{12}N_{21}$. The first term $N_{11}N_{22}$ can be viewed as a bi-dimensional measure of both richly-endowed/high-technology countries and poorly-endowed/low-technology countries while the second term $N_{12}N_{21}$ can be viewed as a bi-dimensional measure of both richly-endowed/low-technology countries and poorly-endowed/high-technology countries.[10]

From a different perspective, the response of the transboundary pollution stock path to a reduction in global inequality results from the combined action of two opposite forces. The mitigating force can be associated with a bi-dimensional measure of both richly-endowed/high-technology countries and poorly-endowed/low-technology countries. The aggravating force can be associated with a bi-dimensional measure of both richly-endowed/low-technology countries and poorly-endowed/high-technology countries. Analyzing both equations (64) and (58) shows that the difference between the two bi-dimensional measures is a crucial factor for understanding the response of the path of the transboundary pollution stock to a reduction in global inequality.

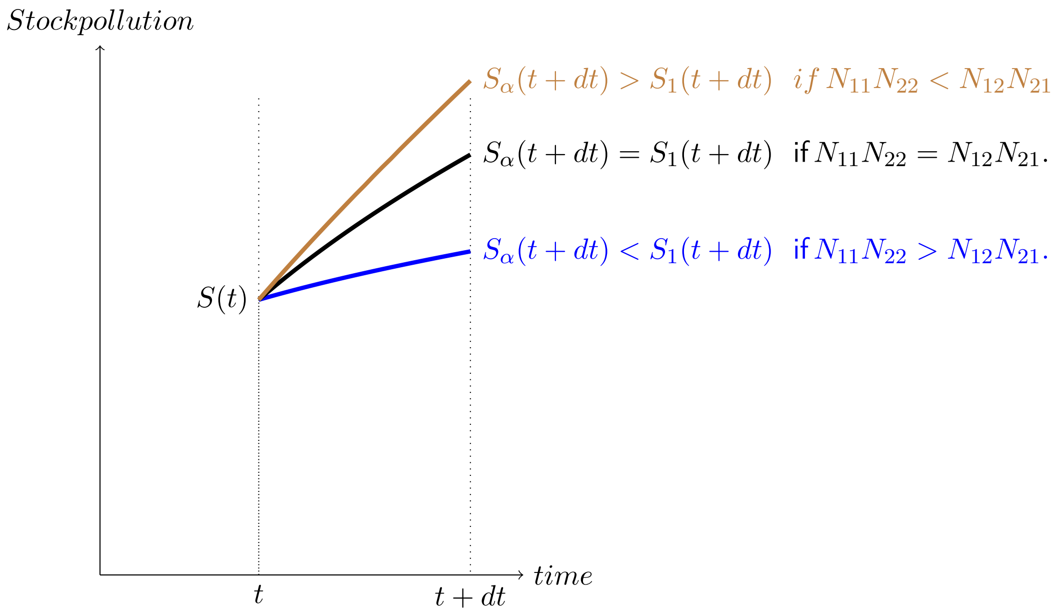

Figure 5: Response of the transboundary pollution stock path to a reduction in global inequality: Bi-dimensional measure explanation

Figure 5 depicts plausible responses of the transboundary pollution stock path to a reduction in global inequality as it relates to bi-dimensional measure of countries. If we assume that the bi-dimensional measure $N_{12}N_{21} = 0$ and the bi-dimensional measure $N_{11}N_{22} \neq 0$, which means that only both poorly-endowed/low-technology countries and richly-endowed/high-technology countries influence the path of the transboundary pollution stock, then the prediction is that a reduction in global inequality would mitigate the pollution stock trajectory [The interpretation reduces to that of equation (45c)]. In contrast, if we assume that the bi-dimensional measure $N_{12}N_{21} \neq 0$ and the bi-dimensional measure $N_{11}N_{22} = 0$, which means that only both poorly-endowed/high-technology countries and richly-

[10]It is worth mentioning that the idea of bi-dimensional measure that shows up in this paper [equation 58)] can be thought as a special case of the concept of product measure on product spaces discussed in the measure theory and integration literature [see for instance Stokey and Lucas (1989, chap 7)].

endowed/low-technology countries influence the path of the transboundary pollution stock, then the prediction is that a reduction in global inequality would aggravate the path of the transboundary pollution stock [The interpretaion reduces to that of equation (45a)]. Therefore in a general case, where both bi-dimensional measures $N_{11}N_{22} \neq 0$ and $N_{12}N_{21} \neq 0$, it becomes clear that there are two forces that work in opposite directions in determining the direction of the transboundary pollution stock path as a result of a reduction in global inequality. The mitigating force is associated with the bi-dimensional measure $N_{11}N_{22}$ of richly-endowed/high-technology countries and poorly-endowed/low-technology countries. The aggravating force is associated with the bi-dimensional measure $N_{12}N_{21} \neq 0$ of richly-endowed/low-technology countries and poorly-endowed/high-technology countries. Therefore, the pollution stock will follow a path below the "Business-as-usual" scenario path as a response to a reduction in global inequality if the mitigating force associated with the bi-dimensional measure $N_{11}N_{11}$ more than offsets the aggravating force associated with the bi-dimensional measure $N_{12}N_{21}$.

6 Concluding remarks

Global inequality has been a major point of concern at almost every international conference related to climate change issues. In this paper, we have explored some conditions under which reducing global inequality is likely to produce a mitigating effect on the path of the future transboundary pollution stock. The response of the path of the international transboundary pollution stock to a reduction in global inequality is analyzed through three interrelated approaches: A Ratio rich-poor approach, an average endowment approach, and a bi-dimensional measure approach.

From the ratio rich-poor approach, a reduction in global inequality from richly-endowed countries in favor of poorly-endowed countries would have a mitigating effect on the path of the transboundary pollution stock if the relative number of richly endowed countries in the subgroup of high-tech countries is greater than the relative number of richly-endowed countries in the subgroup of low-tech countries. If the relative number of richly endowed countries in the subgroup of high-tech countries is equal to the relative number of richly-endowed countries in the subgroup of low-tech countries, a reduction in global inequality will have a neutral effect on the path of the transboundary pollution stock. A reduction in global inequality from richly-endowed countries in favor of poorly-endowed countries would have an aggravating effect on the path of the transboundary pollution stock if the relative number of richly endowed countries in the subgroup of high-tech countries is smaller that the relative number of richly-endowed countries in the subgroup of low-tech countries.

From the average endowment approach, a reduction in global inequality would have a mitigating effect on the path of the transboundary pollution stock if the subgroup of high-technology countries is in average richer than the subgroup of low-technology countries after a reduction in global inequality.

From the bi-dimensional measure approach, the response of the transboundary pollution stock path to a reduction in global inequality results from the combined action of two opposite forces. The mitigating force can be associated with a bi-dimensional measure of both richly-endowed/high-technology countries and poorly-endowed/low-technology countries. The aggravating force can be associated with a bi-dimensional measure of both richly-endowed/low-technology countries and poorly-endowed/high-technology countries.

From an international political economy perspective, this paper suggests that the effectiveness of the reduction of global inequality as a mean of mitigating climate change is tied to the "validation criteria" that would be used for classifying countries in terms of both technology and non-polluting endowment. Such a set of criteria has important implications since countries ascribed as rich countries would be required to make financial transfers to countries ascribed as poor countries in order to reduce global inequality. As derived from this paper, reducing global inequality will have a mitigating effect on the path of the transboundary pollution stock if the relative number of richly-endowed countries in the subgroup of high-tech countries is greater that the relative number of richly-endowed countries in the subgroup of low-tech countries. From an applied policy perspective, this result provides a guidance tool that may prove useful in designing international and implementing redistributive policies aimed at mitigating the climate change problem. Finally, the predictions brought out by this paper may prove useful for empirical studies on issues related to global inequality reduction and transboundary pollution. This is left for future research.[11]

[11] Greater availability of Big Data on the world scale will prove useful for this task.

Appendix

A Computing equilibrium emission strategies

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